

## Key Problems

### Problem 1.

Determine the definiteness of the symmetric matrix:

$$\text{a) } A = \begin{pmatrix} 7 & 4 \\ 4 & 3 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} -1 & 1 \\ 1 & -3 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

$$\text{d) } A = \begin{pmatrix} 2 & 3 & -5 \\ 3 & 7 & 0 \\ -5 & 0 & 35 \end{pmatrix}$$

$$\text{e) } A = \begin{pmatrix} -1 & -2 & -2 \\ -2 & -4 & -4 \\ -2 & -4 & -2 \end{pmatrix}$$

### Problem 2.

Find the symmetric matrix of the quadratic form, and determine its definiteness:

$$\text{a) } f(x,y) = x^2 - 8xy + 3y^2$$

$$\text{b) } f(x,y,z) = 2x^2 - 2xz + 3y^2 + z^2$$

$$\text{c) } f(x,y,z) = 3x^2 + 4xy - 4xz + 3y^2 + 4yz + 8z^2$$

$$\text{d) } f(x,y,z,w) = xw - yz$$

### Problem 3.

Determine the definiteness of the symmetric matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

### Problem 4.

Find an orthogonal matrix  $P$  such that  $P^T A P = D$  is a diagonal matrix:

$$\text{a) } A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

### Problem 5.

Find the equilibrium state  $\mathbf{v}$  of the Markov chains with transition matrix  $A$ . Is the Markov chain regular?

$$\text{a) } A = \begin{pmatrix} 0.30 & 0.15 \\ 0.70 & 0.85 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} 0.86 & 0.42 \\ 0.14 & 0.58 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 0.75 & 0.02 & 0.10 \\ 0.20 & 0.90 & 0.20 \\ 0.05 & 0.08 & 0.70 \end{pmatrix}$$

$$\text{d) } A = \begin{pmatrix} 1 & 0.3 \\ 0 & 0.7 \end{pmatrix}$$

$$\text{e) } A = \begin{pmatrix} 0.2 & 0.4 & 0 \\ 0.8 & 0.4 & 0.7 \\ 0 & 0.2 & 0.3 \end{pmatrix}$$

## Exercise Problems

Problems from the textbook: [E] 4.8 - 4.17

Exam problems:

Midterm exam 10/2018 Question 1-7

Midterm exam 01/2019 Question 1-7

## Answers to Key Problems

### Problem 1.

- a) Positive definite                      b) Negative definite                      c) Positive definite  
d) Positive semi-definite                e) Indefinite

### Problem 2.

- a) Indefinite                                b) Positive definite                      c) Positive semi-definite  
d) Indefinite                                e) Indefinite

### Problem 3.

Positive semi-definite

### Problem 4.

a)  $P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$                       b)  $P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$                       c)  $P = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{3} & 1 & \sqrt{2} \\ 0 & -2 & \sqrt{2} \\ \sqrt{3} & 1 & \sqrt{2} \end{pmatrix}$

### Problem 5.

a)  $\mathbf{v} = \begin{pmatrix} 3/17 \\ 14/17 \end{pmatrix}$  (regular)                      b)  $\mathbf{v} = \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}$  (regular)                      c)  $\mathbf{v} = \begin{pmatrix} 2/15 \\ 10/15 \\ 3/15 \end{pmatrix}$  (regular)  
d)  $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  (not regular)                      e)  $\mathbf{v} = \begin{pmatrix} 7/25 \\ 14/25 \\ 4/25 \end{pmatrix}$  (regular)