Key Problems

Problem 1.
Solve the difference equations:

a) $y_{t+1} - 6y_t = 10t + 3$

b) $y_{t+2} - 5y_{t+1} + 6y_t = 2t$

c) $y_{t+2} - 4y_{t+1} + 4y_t = 1$

d) $y_{t+2} + y_{t+1} - 2y_t = 6$

Problem 2.

a) Solve the difference equation $x_{t+1} = 3x_t + 4,\ x_0 = 2$ and compute x_5.

b) You borrow an amount K. The interest rate per period is r. The repayment is 500 in the first period, and increases with 10 for each subsequent period. Show that the outstanding balance b_t after period t satisfies the difference equation

$$b_{t+1} = (1 + r)b_t - (500 + 10t), \quad b_0 = K$$

and solve this difference equation.

Problem 3.
We consider a model for housing prices, where p_t is the price after t years. The model is given by the difference equation

$$p_{t+2} - 2p_{t+1} + p_t = -15, \quad p_0 = 695,\ p_1 = 743$$

a) Solve the difference equation.

b) We define $d_t = p_{t+1} - p_t$ to be the change in housing prices. Show that $d_{t+1} - d_t$ is constant, and use this to determine when housing prices will increase and when housing prices will decrease.

Problem 4.
Solve the systems of difference equations:

a) $\mathbf{y}_{t+1} = \begin{pmatrix} -5 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & -5 \end{pmatrix} \cdot \mathbf{y}_t, \quad \mathbf{y}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

b) $\mathbf{y}_{t+1} = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 3 & -1 & 1 \end{pmatrix} \cdot \mathbf{y}_t, \quad \mathbf{y}(0) = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

Exercise Problems

Problems from the textbook	[E] 8.1 - 8.9, 9.8
Final exam problems	Final exam 11/2019 Q5, 01/2021 Q3a, 03/2021 Q3a

Answers to Key Problems

Problem 1.

a) $y_t = C \cdot 6^t - 2t - 1$

b) $y_t = C_1 \cdot 2^t + C_2 \cdot 3^t + t + 3/2$

c) $y_t = (C_1 + C_2 t) \cdot 2^t + 1$

d) $y_t = C_1 + C_2 \cdot 2^t + 2t$

Problem 2.

a) $x_t = 4 \cdot 3^t - 2,\ x_5 = 970$

b) $b_t = \left(K - \frac{10}{r^2} - \frac{500}{r}\right)(1+r)^t + \frac{10}{r}t + \frac{10}{r^2} + \frac{500}{r}$

Problem 3.

a) $p_t = 695 + 55.5t - 7.5t^2$

b) $d_{t+1} - d_t = -15$, $d_t > 0$ for $t = 0, 1, 2, 3$ and that $d_t < 0$ for $t \geq 4$

Problem 4.

a) $\mathbf{y}_t = \frac{1}{2}\begin{pmatrix}1\\0\\1\end{pmatrix} \cdot (-4)^t - \frac{1}{2}\begin{pmatrix}-1\\0\\1\end{pmatrix} \cdot (-6)^t$

b) $\mathbf{y}_t = \begin{pmatrix}0\\-1\\1\end{pmatrix} \cdot 2^t + \begin{pmatrix}1\\-1\\2\end{pmatrix} \cdot 3^t$