

## Key Problems

### Problem 1.

Write the systems of differential equations on matrix form and solve them:

$$\text{a) } \begin{cases} y_1' = 2y_1 + 5y_2 \\ y_2' = 5y_1 + 2y_2 \end{cases}$$

$$\text{b) } \begin{cases} y_1' = y_2 \\ y_2' = 4y_1 + 3y_2 \end{cases}$$

$$\text{c) } \begin{cases} y_1' = y_1 + 4y_2 + 3 \\ y_2' = y_1 - 2y_2 - 3 \end{cases}$$

### Problem 2.

Solve the systems of difference equations:

$$\text{a) } \mathbf{y}' = \begin{pmatrix} -5 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & -5 \end{pmatrix} \cdot \mathbf{y}, \quad \mathbf{y}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{b) } \mathbf{y}' = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 3 & -1 & 1 \end{pmatrix} \cdot \mathbf{y}, \quad \mathbf{y}(0) = \begin{pmatrix} -1 \\ -3 \\ 8 \end{pmatrix}$$

### Problem 3.

Solve the differential equation  $y''' + 4y'' + y' - 6y = 0$ .

### Problem 4.

Let  $y(t) = 3e^{-2t} - 5e^t + 12e^{-3t}$ .

- Find a linear second order differential equation that has  $y$  as a particular solution.
- Find a linear third order differential equation that has  $y$  as a particular solution.
- Find a  $3 \times 3$  matrix  $A$  such that  $\mathbf{y}' = A\mathbf{y}$  has  $\mathbf{y} = (y, y', y'')$  as a particular solution.

### Problem 5.

Find the equilibrium states and determine their stability:

$$\text{a) } y'' + 5y' + 6y = 0$$

$$\text{b) } y'' + 5y' + 6y = 12$$

$$\text{c) } y'' + y' - 20y = 1$$

$$\text{d) } y''' + 4y'' + y' - 6y = 0$$

## Exercise Problems

Problems from the textbook [E] 9.1 - 9.7

Final exam problems 11/2018 Q2,Q5, 01/2019 Q2, 01/2020 Q3, 03/2021 Q3bc

## Answers to Key Problems

### Problem 1.

$$\text{a) } \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} C_1 e^{7t} - C_2 e^{-3t} \\ C_1 e^{7t} + C_2 e^{-3t} \end{pmatrix} \quad \text{b) } \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} C_1 e^{4t} - C_2 e^{-t} \\ 4C_1 e^{4t} + C_2 e^{-t} \end{pmatrix} \quad \text{c) } \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4C_1 e^{2t} - C_2 e^{-3t} + 1 \\ C_1 e^{2t} + C_2 e^{-3t} - 1 \end{pmatrix}$$

### Problem 2.

$$\text{a) } \mathbf{y} = \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-4t} - \frac{1}{2} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-6t} \quad \text{b) } \mathbf{y} = \begin{pmatrix} -2 \\ -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} e^{3t}$$

### Problem 3.

$$y = C_1 \cdot e^t + C_2 \cdot e^{-2t} + C_3 e^{-3t}$$

### Problem 4.

More than one solution is possible:

$$\text{a) } y'' + y' - 2y = 48e^{-3t} \quad \text{b) } y''' + 4y'' + y' - 6y = 0 \quad \text{c) } A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -1 & -4 \end{pmatrix}$$

### Problem 5.

- a)  $\mathbf{y}_e = (0,0)$  is globally asymptotically stable      b)  $\mathbf{y}_e = (2,0)$  is globally asymptotically stable  
c)  $\mathbf{y}_e = (-1/20,0)$  is unstable      d)  $\mathbf{y}_e = (0,0,0)$  is unstable