

Your answers should be provided as a single file in pdf format, and a single attachment in txt format. The attachment should contain python code, and the main file should contain the rest of your answer. It is almost always best to write mathematics by hand and scan to pdf.

**You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.**

### Question 1.

The python code below defines a function `f`. Explain what this function does when called with a positive integer `x` as argument.

```
def f(x):
    # print(x)
    if x == 1:
        return(1)
    elif x % 2 == 0:
        return f(x//2)+1
    else:
        return f(3*x+1)+1
```

### Question 2.

- (a) Find all complex solutions of  $x^6 = 64$ .
- (b) Find all complex solutions of  $x^6 + x^3 + 1 = 0$ .
- (c) Find all complex eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & -1 & -3 \\ 2 & 3 & 1 \\ 3 & 2 & -2 \end{pmatrix}$$

and use this to determine whether it has any real eigenvectors.

### Question 3.

Write python code that defines a function `reduced(matrix)`. When called with a NumPy array `matrix`, it should return the reduced echelon form of `matrix`. Executable python code that defines this function should be included as a txt-file attachment, and runtime examples (for example printouts from Jupyter) should be included in your pdf file, showing that your code can find the reduced echelon form of the following matrices:

```
A = np.array([[1,1,1,3,-1],[1,2,4,7,3],[2,3,5,10,2]])
B = np.array([[1,3,1],[1,4,3],[2,3,5],[-1,10,2]])
```