Plan

1. About this year's final exam
2. Final exam 11/2019 Q1 - Q4
3. Final exam 01/2021 Q2, Q3b

1 About this year's final exam

(a) Topics emphasized this year:

i) Use of matrices in optimization problems

$f(\underline{x}) = \underline{x}^T A \underline{x}$ quadr. form $\qquad g(\underline{x}) = B\underline{x}$ linear fcns

$f'(\underline{x}) = 2A\underline{x} \qquad g'(\underline{x}) = B^T$

$H(f) = 2A \qquad H(g) = 0$

Ex: $f(x,y,z) = \underbrace{x^2+2xy+3y^2+z^2}+\underbrace{x-y}$

$$= (x\ y\ z)\underbrace{\begin{pmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \underbrace{(1\ \ -1\ \ 0)}_{B}\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$f'(\underline{x}) = 2A\underline{x} + B^T = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 6 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2x+2y+1 \\ 2x+6y-1 \\ 2z \end{pmatrix} \begin{matrix} \leftarrow f'_x \\ \leftarrow f'_y \\ \leftarrow f'_z \end{matrix}$$

$$H(f) = 2A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 6 & 0 \\ 0 & 0 & 2 \end{pmatrix} \longleftarrow$$ A and H(f) has the same definiteness

ii) Orthogonal diagonalization:

A symmetric $\Longrightarrow$ $P^{-1}AP = D$ diagonalization such that $P^{-1} = P^T$ orthogonal ($v_i \cdot v_i = 1$, $v_i \cdot v_j = 0$ $i \neq j$)

$$\boxed{\underset{\text{quadr. form}}{f(\underline{x}) = \underline{x}^T A \underline{x}} = \lambda_1 \cdot u_1^2 + \lambda_2 u_2^2 + \dots + \lambda_n u_n^2}$$

(where $\lambda_1, \dots, \lambda_n$ eigenvalues of A, and $x = P \cdot \underline{u}$ change of variables)

(b) The final exam: 12+1 questions 72p = 100%
looks like early school exams

Preparations: Do exam problems

TA session: Monday 17-19 D1-065, C1-067
Office hours:

Course evaluation:

② Exam 11/2019:

1.

a)
$$A = \begin{pmatrix} 2 & 1 & 5 & 9 \\ -1 & 1 & 2 & -3 \\ 3 & 0 & 1 & 10 \\ 0 & 3 & 0 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 7 & 6 \\ -1 & 1 & 2 & -3 \\ 3 & 0 & 1 & 10 \\ 0 & 3 & 0 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 7 & 6 \\ 0 & 3 & 9 & 3 \\ 0 & -6 & -20 & -8 \\ 0 & 3 & 0 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 7 & 6 \\ 0 & 3 & 9 & 3 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & -9 & -9 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 7 & 6 \\ 0 & 3 & 9 & 3 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\operatorname{rk} A = \underline{\underline{3}}$

b) $\dim \operatorname{Null}(A) = n - \operatorname{rk}(A)$
$= 4 - 3 = \underline{\underline{1}}$

Base: $A\underline{x} = \underline{0}$

$x + 2y + 7z + 6w = 0$
$3y + 9z + 3w = 0$
$-2z - 2w = 0$

$x + 2(2w) + 7(-w) + 6w = 0$, $\underline{x = -3w}$

$3y + 9(-w) + 3w = 0$, $\frac{3y}{3} = \frac{6w}{3}$, $\underline{y = 2w}$

$\underline{z = -w}$

$\underline{w \text{ free}}$

$$\underline{x} = \begin{pmatrix} -3w \\ 2w \\ -w \\ w \end{pmatrix} = w \cdot \begin{pmatrix} -3 \\ 2 \\ -1 \\ 1 \end{pmatrix} \Rightarrow \underline{w} = \begin{pmatrix} -3 \\ +2 \\ -1 \\ 1 \end{pmatrix}$$

base at Null(A)

c) Find v_4: $\underline{w}$ in Null(A) $\Rightarrow -3\underline{v_1} + 2\underline{v_2} - \underline{v_3} + \underline{v_4} = \underline{0}$

$A \cdot \underline{w} = \underline{0}$

$\underline{\underline{\underline{v_4} = 3\underline{v_1} - 2\underline{v_2} + \underline{v_3}}}$

2.
$$A=\begin{pmatrix}4&0&6\\-1&3&0\\1&1&2\end{pmatrix} \qquad \underline{v}=\begin{pmatrix}-3\\-1\\2\end{pmatrix}$$

a) $A\cdot\underline{v}=\lambda\cdot\underline{v} \iff \underline{v}\neq\underline{0}$ eigenvector of A with eigenvalue of A

$$A\underline{v}=\begin{pmatrix}4&0&6\\-1&3&0\\1&1&2\end{pmatrix}\begin{pmatrix}-3\\-1\\2\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}=\lambda\cdot\begin{pmatrix}-3\\-1\\2\end{pmatrix}$$

$\underline{v}$ is an eigenvector $\Leftarrow$ holds for $\lambda=0$

b) Eigenvalues of A: $\begin{vmatrix}4-\lambda&0&6\\-1&3-\lambda&0\\1&1&2-\lambda\end{vmatrix}=0$

$$(3-\lambda)\cdot[(4-\lambda)(2-\lambda)-6]-(-1)\cdot[0-6]$$
$$=(3-\lambda)\left((4-\lambda)(2-\lambda)-6\right)-6=0$$
$$(3-\lambda)(\lambda^2-6\lambda+2)-6=0$$
$$3\lambda^2-\lambda^3-18\lambda+6\lambda^2+\cancel{6}-2\lambda-\cancel{6}=0$$
$$-\lambda^3+9\lambda^2-20\lambda=0$$
$$-\lambda(\lambda^2-9\lambda+20)=0$$
$$\lambda=0 \quad \lambda=4,\ \lambda=5$$

Alt:
$$(4-\lambda)(3-\lambda)(2-\lambda)+6\cdot(-1-(3-\lambda))=0$$
$$(4-\lambda)(3-\lambda)(2-\lambda)+6(\lambda-4)=0$$
$$(4-\lambda)\cdot[\lambda^2-5\lambda+\cancel{6}-\cancel{6}]=0$$
$$(4-\lambda)\cdot\lambda(\lambda-5)=0$$
$$\lambda=4 \quad \lambda=0 \quad \lambda=5$$

c) A is diagonalizable since A is 3×3 and we have three distinct eigenvalues.

3. a) $y' - 4y = 10e^{-t}$ linear first order

$$y = y_h + y_p = Ce^{4t} - 2e^{-t}$$

y_h: $y' - 4y = 0$

$r - 4 = 0$

$r = 4$ $\quad y_h = C \cdot e^{4t}$

y_p: $y = Ae^{-t}$

$y' = -Ae^{-t}$

$(-Ae^{-t}) - 4(Ae^{-t}) = 10e^{-t}$

$-5Ae^{-t} = 10e^{-t}$

$-5A = 10$

$A = -2$ $\quad y_p = -2e^{-t}$

Alt:

$a(t) = -4$

$\int a(t) = -4t + C \Rightarrow u = e^{-4t}$

$(y \cdot e^{-4t})' = 10e^{-t} \cdot e^{-4t} = 10e^{-5t}$

$$\frac{y \cdot e^{-4t}}{e^{-4t}} = \int 10e^{-5t}\,dt = \frac{-2e^{-5t} + C}{e^{-4t}}$$

$$y = (-2e^{-5t} + C)e^{4t}$$

$$= -2e^{-t} + Ce^{4t}$$

b) $2t + 2ty^2 + (2y + 2yt^2)y' = 0$

$h'_t = 2t + 2ty^2 \Rightarrow h = t^2 + t^2y^2 + \alpha(y)$

$h'_y = 2y + 2yt^2$

$h'_y = \cancel{0} + \cancel{t^2 \cdot 2y} + \alpha'(y) = 2y + \cancel{2yt^2}$

$\alpha(y) = y^2$

The eq is exact, and $\Leftarrow h = t^2 + t^2y^2 + y^2$

$t^2 + t^2y^2 + y^2 = C$

$(t^2+1)y^2 = C - t^2$

$$y^2 = \frac{C - t^2}{t^2+1} \Rightarrow y = \pm\sqrt{\frac{C-t^2}{t^2+1}}$$

c) $\underline{y}' = \begin{pmatrix} 4 & 0 & 6 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{pmatrix} \cdot \underline{y}$ ← Same matrix as in $\underline{2}$ $\Rightarrow \lambda_1 = 0, 4, 5$ from $\underline{2\,b)}$.

General solution:

$$\underline{y} = C_1 \cdot \underline{v_1} e^{\lambda_1 t} + C_2 \cdot \underline{v_2} e^{\lambda_2 t} + C_3 \underline{v_3} e^{\lambda_3 t}$$

$$= C_1 \cdot \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} e^{0} + C_2 \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} e^{4t} + C_3 \begin{pmatrix} 6 \\ -3 \\ 1 \end{pmatrix} e^{5t}$$

$$= C_1 \cdot \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} + C_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} e^{4t} + C_3 \begin{pmatrix} 6 \\ -3 \\ 1 \end{pmatrix} e^{5t}$$

where $\lambda_1 = 0$, $\lambda_2 = 4$, $\lambda_3 = 5$ and $\underline{v_i}$ bases of E_{λ_i}

$\underline{E_0}$: $\underline{v_1} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$ from $2\underline{a}$.

$\underline{E_4}$: $\begin{pmatrix} 0 & 0 & 6 \\ -1 & -1 & 0 \\ 1 & 1 & -2 \end{pmatrix} \xrightarrow{2} \begin{pmatrix} -1 & -1 & 0 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{pmatrix}$

$-x - y = 0$
$6z = 0$
y free

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y \\ y \\ 0 \end{pmatrix} = y \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ $\quad \underline{v_2} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$\underline{E_5}$: $\begin{pmatrix} -1 & 0 & 6 \\ -1 & -2 & 0 \\ 1 & 1 & -3 \end{pmatrix} \xrightarrow{\ \ \cdot(-1)} \begin{pmatrix} -1 & 0 & 6 \\ 0 & -2 & -6 \\ 0 & 0 & 0 \end{pmatrix}$

$-x \quad + 6z = 0$
$-2y - 6z = 0$
z free

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6z \\ -3z \\ z \end{pmatrix} = z \begin{pmatrix} 6 \\ -3 \\ 1 \end{pmatrix}$ $\quad \underline{v_3} = \begin{pmatrix} 6 \\ -3 \\ 1 \end{pmatrix}$

4. $f(x,y,z) = x^2+y^2+z^2 - xy + xz - yz$

a) $= \underline{x}^T A \underline{x}$, $A = \begin{pmatrix} 1 & -1/2 & 1/2 \\ -1/2 & 1 & -1/2 \\ 1/2 & -1/2 & 1 \end{pmatrix}$

$\left(\begin{array}{l} \text{Of } A: \\ D_1 = 1 \\ D_2 = 3/4 \\ D_3 = 4/8 \end{array} \right)$

Of $H(f)$:

$D_1 = 2$

$D_2 = \underline{3}$

$D_3 = 1\cdot(-1) - (-1)(-1) + 2\cdot 3$

$= -1 - 1 = 6 = \underline{4}$

$H(f) = 2A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

$H(f)$ / A is pos. defn.

$\Downarrow$

f is convex

$x+y+z = 11$

b) min $f(\underline{x}) = \underline{x}^T A \underline{x}$ wh $B\underline{x} = 11$

$B = (1 \ 1 \ 1)$

$\Downarrow$

$L = \underline{x}^T A \underline{x} - \lambda(B\underline{x} - 11)$

FOC: $L'(\underline{x}) = \boxed{\begin{array}{l} 2A\underline{x} - \lambda(B^T) = \underline{0} \\ B\underline{x} = 11 \end{array}}$

C:

$$2A\underline{x} = \lambda B^T$$
$$B\underline{x} = 11$$

$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda \\ \lambda \\ \lambda \end{pmatrix}$

$(1 \ 1 \ 1) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (11)$

$\boxed{\begin{array}{rl} 2x - y + z & = \lambda \\ -x + 2y - z & = \lambda \\ x - y + 2z & = \lambda \\ x + y + z & = 11 \end{array}}$

$\Downarrow$

$\left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 0 \\ -1 & 2 & -1 & -1 & 0 \\ 1 & -1 & 2 & -1 & 0 \\ 1 & 1 & 1 & 0 & 11 \end{array} \right) \leftarrow \left(\begin{array}{cccc|c} 2 & -1 & 1 & -1 & 0 \\ -1 & 2 & -1 & -1 & 0 \\ 1 & -1 & 2 & -1 & 0 \\ 1 & 1 & 1 & 0 & 11 \end{array} \right) \Leftarrow \boxed{\begin{array}{rl} 2x - y + z - \lambda & = 0 \\ -x + 2y - z - \lambda & = 0 \\ x - y + 2z - \lambda & = 0 \\ x + y + z & = 11 \end{array}}$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 0 \\ 0 & 3 & -1 & -3 & 0 \\ 0 & -2 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 11 \end{array}\right)]_1 \to \left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & -2 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 11 \end{array}\right)]2$$

$$\to \left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 4 & -3 & 0 \\ 0 & 0 & 1 & 2 & 11 \end{array}\right) \updownarrow \to \left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 2 & 11 \\ 0 & 0 & 4 & -3 & 0 \end{array}\right)]-4$$

$$\to \left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 2 & 11 \\ 0 & 0 & 0 & -11 & -44 \end{array}\right)$$

$x + 5 - 2(4) = 0$ $\quad \underline{x = 3}$

$y + 3 - 2 \cdot 4 = 0$ $\quad \underline{y = 5}$

$z + 2 \cdot 4 = 11$ $\quad \underline{z = 3}$

$-11\lambda = -44 \Rightarrow \underline{\lambda = 4}$

Cand. pts: $(x,y,z;\lambda) = \underline{(3,5,3;4)}$ $\qquad x+y+z-11$

Use SOC: $h = L(x,y,z;4) = \underline{x}^T A \underline{x} - 4(B\underline{x} - 11)$

$H(h) = 2A$ pos. defn. from a)

$\Rightarrow h$ convex $\underset{SOC}{\Rightarrow}$ $f_{min} = f(3,5,3) = \underline{22}$ with $\boxed{\lambda = 4}$

c) Change constraint: $x+y+z = 11 \to \boxed{x+y+z=10}$

Lagrange pb. with parameter a: $\quad$ min $f(x,y,z)$ when $x+y+z-a=0$

$L = f(x,y,z) - \lambda(x+y+z-a) = \ldots + \lambda a$

$f^*(11) = 22$ in (b). $\quad \Rightarrow L'_a = \lambda \underset{\text{Env. thm.}}{\Rightarrow} \dfrac{d f^*(a)}{da} = L'_a(\underline{x}^*(a); \lambda^*(a))$

$= \lambda^*(a) = 4$ from (b) $\quad (a=11)$

$$f^*(10) \simeq \underbrace{f^*(11)}_{22} + \underbrace{\Delta a}_{10-11=-1} \cdot \underbrace{\frac{df^*(a)}{da}}_{4} \approx 22 - 1 \cdot 4 = \underline{18}$$

③ Final exam 01/2021:

i) linear ?
ii) sep. ?
iii) exact ?

3b) $t^2y' + 2ty = 1$

Linear: $y' + \frac{2t}{t^2}y = \frac{1}{t^2}$ Yes

$y' + \frac{2}{t}y = \frac{1}{t^2}$

$\int \frac{2}{t}\,dt = 2\ln|t| + C$

$u = e^{2\ln|t|}$

$= (e^{\ln|t|})^2$

$= |t|^2 = t^2$

$t^2y' + 2t\cdot y = 1$

$(t^2\cdot y)' = 1$

$t^2y = t + C$

$y = \frac{t+C}{t^2}$

Sep: $t^2y' = 1 - 2ty$

$y' = \frac{1-2ty}{t^2} = f(t)\cdot g(y)$? not possible ⇓ not sep.

Exact: $t^2y' + 2ty = 1$

$2ty - 1 + t^2y' = 0$

$h'_t = 2ty - 1$ $\quad (t^2y + a(t))'_t = \cancel{2ty} + a'(t) = \cancel{2ty} - 1$

$h'_y = t^2 \quad \Rightarrow \quad h = t^2y + a(t) \quad a(t) = -t$

Yes exact : $h = t^2y - t = C$

$\frac{t^2y}{t} = \frac{C+t}{t^2}$

$y = \frac{C+t}{t^2}$

2\. a) $f(x,y,z) = \ln(5 - x^2 + xy - y^2 + yz + z^2 - xz)$

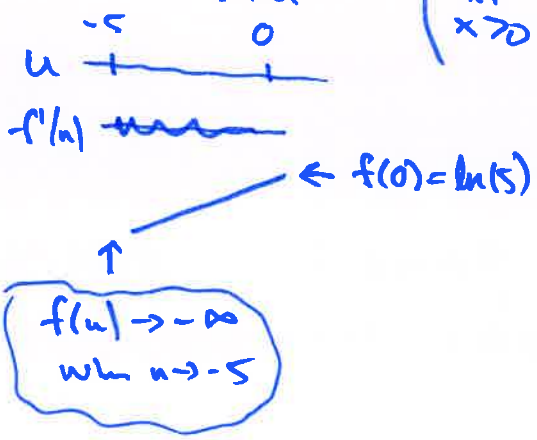
$= \ln(5+u)$, $u = -x^2 + xy - y^2 + yz + z^2 - xz$

Inner fn: $u = \underline{x}^T A \underline{x}$ quadr. fn $A = \begin{pmatrix} -1 & 1/2 & -1/2 \\ 1/2 & -1 & 1/2 \\ -1/2 & 1/2 & -1 \end{pmatrix}$

Outer fn:

$f(u) = \ln(5+u)$, $D_u = (-5, 0]$

$f'(u) = \frac{1}{5+u} \cdot 1 = \frac{1}{5+u} > 0$ (since $\ln(x)$ is defn. for $x > 0$)



$f_{max} = \underline{\underline{\ln(5)}}$ at $u = 0$

$H(u) = 2A = \begin{pmatrix} -2 & 1 & -1 \\ 1 & -2 & 1 \\ -1 & 1 & -2 \end{pmatrix}$

$D_1 = \underline{-2}$

$D_2 = \underline{3}$

$D_3 = -2 \cdot 3 - 1 \cdot (-1) - 1(-1) = -6 + 2 = \underline{-4}$

$\underline{u \text{ concave}} \Leftarrow H(u)$ neg. defn.

Stat. pts of u = global max of u:

$u'(\underline{x}) = \underline{0}$

$2A\underline{x} = 0 \quad | \cdot \frac{1}{2}$

$A\underline{x} = 0 \quad | A^{-1} \cdot$

$\underline{x} = A^{-1}\underline{0} = \underline{0}$

$\Rightarrow \underline{x} = \underline{0}$

$(x,y,z) = (0,0,0)$ stat. pts

$\Downarrow$

$u_{max} = u(0,0,0) = \underline{0}$

$u \le 0$

b) $D: xw + yz \le -2$

D is closed ($\le$)

Is D bounded?

(compact = closed + bounded)

$x = w = 0$, $y = a$, $z = -a$ where a is a parameter

$\Downarrow$

$(0, a, -a, 0)$ is in D $\Leftrightarrow$ $0 \cdot 0 + a(-a) = -a^2 \le -2$

(holds if $a^2 \ge 2$)

$\Downarrow$

there are points in D with $y \to \infty$, $z \to -\infty$, and this means that D is not bounded

(for example, $a = 100$: $100^2 \ge 2$ ok $\Rightarrow$ $(0, 100, -100, 0)$ is in D

$1000^2 \ge 2$ ok $(0, 1000, -1000, 0)$ is in D

etc.)

Alt method: $xw + yz = \underline{x}^T A \underline{x}$ with $A = \begin{pmatrix} 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \end{pmatrix}$

A symmetric $\Rightarrow$

A has orthogonal diag:

$\underline{x}^T A \underline{x} = \lambda_1 u_1^2 + \lambda_2 u_2^2 + \lambda_3 u_3^2 + \lambda_4 u_4^2$

$= \frac{1}{2} u_1^2 + \frac{1}{2} u_2^2 - \frac{1}{2} u_3^2 - \frac{1}{2} u_4^2$

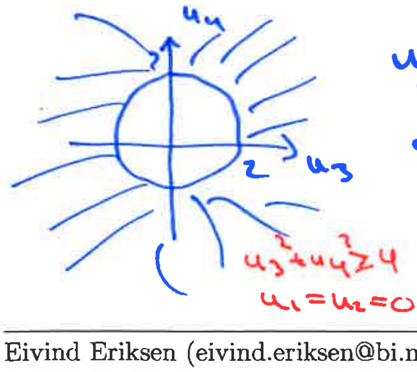
where u_i are linear expr. in x, y, z, w.

We see that $u_1 = u_2 = 0$ means that

$xw + yz = -\frac{1}{2} u_3^2 - \frac{1}{2} u_4^2 \le -2$

$\Updownarrow$

$u_3^2 + u_4^2 \ge 4$



see that u_3, u_4 can go towards ∞

A has both positive and negative eigenvalues:

Alt i): $\Delta_2 = M_{23,23} = \begin{vmatrix} 0 & 1/2 \\ 1/2 & 0 \end{vmatrix}$

$= -1/4 < 0$

$\Rightarrow$ A indef.

$\Rightarrow$ both pos. and neg. eigenvalues

Alt ii): $\begin{vmatrix} -\lambda & 0 & 0 & 1/2 \\ 0 & -\lambda & 1/2 & 0 \\ 0 & 1/2 & -\lambda & 0 \\ 1/2 & 0 & 0 & -\lambda \end{vmatrix} = 0$

$-\lambda \begin{vmatrix} -\lambda & 1/2 & 0 \\ 1/2 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} - \frac{1}{2} \begin{vmatrix} 0 & -\lambda & 1/2 \\ 0 & 1/2 & -\lambda \\ 1/2 & 0 & 0 \end{vmatrix}$

$= \lambda^2(\lambda^2 - 1/4) - 1/4(\lambda^2 - 1/4)$

$= (\lambda^2 - 1/4)(\lambda^2 - 1/4) = (\lambda + 1/2)(\lambda - 1/2) \cdot (\lambda + 1/2) \cdot (\lambda - 1/2) = 0$

$\lambda_1 = \lambda_2 = 1/2$, $\lambda_3 = \lambda_4 = -1/2$