

# Solutions: Problem set B

1, 2, 4: see python code (on web page)

3.

a)

$$A = \begin{pmatrix} f_1 & f_2 & f_3 & f_4 \\ 0 & p_1 & 0 & 0 \\ 0 & 0 & p_2 & 0 \\ 0 & 0 & 0 & p_3 \end{pmatrix}$$

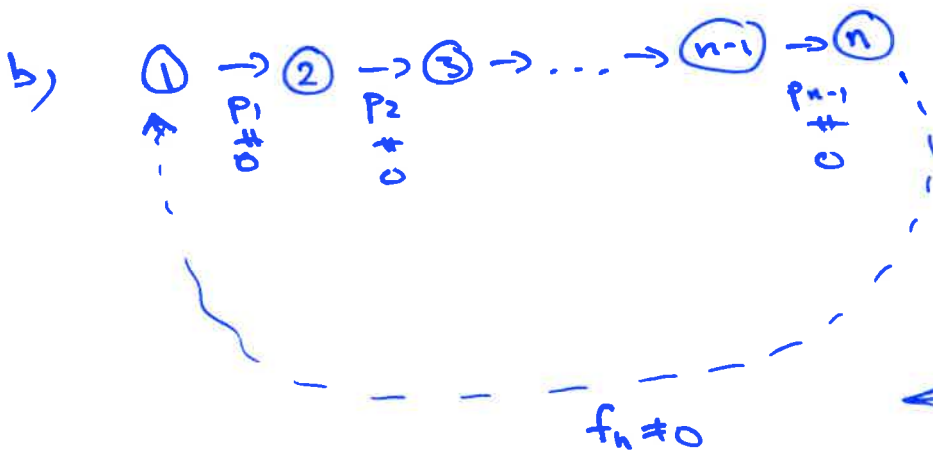
$$\begin{vmatrix} f_1 - \lambda & f_2 & f_3 & f_4 \\ p_1 & -\lambda & 0 & 0 \\ 0 & p_2 & -\lambda & 0 \\ 0 & 0 & p_3 & -\lambda \end{vmatrix} = 0$$

$$-\lambda \cdot \begin{vmatrix} f_1 - \lambda & f_2 & f_3 \\ p_1 & -\lambda & 0 \\ 0 & p_2 & -\lambda \end{vmatrix} - f_4 \begin{vmatrix} p_1 & -\lambda & 0 \\ 0 & p_2 & -\lambda \\ 0 & 0 & p_3 \end{vmatrix} = 0$$

$$-\lambda (-\lambda (-\lambda (f_1 - \lambda) - p_1 f_2) + f_3 (p_1 p_2)) - f_4 (p_1 p_2 p_3) = 0$$

$$\lambda^2 (\lambda^2 - f_1 \lambda - p_1 f_2) + f_3 p_1 p_2 (-\lambda) - f_4 p_1 p_2 p_3 = 0$$

$$\lambda^4 - f_1 \lambda^3 - p_1 f_2 \lambda^2 - p_1 p_2 f_3 \lambda - p_1 p_2 p_3 f_4 = 0$$



If  $f_n = 0$ , then there is no path from node n to node 1.

If  $f_n > 0$ , there is a path

$\Downarrow$   
A is irreducible if and only if  $f_n > 0$ .