

## Key Problems

### Problem 1.

Determine the definiteness of the quadratic form  $f$ :

- a)  $f(x,y,z) = 5x^2 + 6xy + 2y^2 + 16xz + 10yz + 13z^2$     b)  $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + 2xz - 2yw$   
 c)  $f(x,y,z,w) = 2xy + 2xz + 2yw + 2zw$     d)  $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$

### Problem 2.

Determine all values of  $a$  such that the symmetric matrix  $A$  is negative semidefinite:

$$A = \begin{pmatrix} a & 0 & 0 & -1 \\ 0 & a & -1 & 0 \\ 0 & -1 & a & 0 \\ -1 & 0 & 0 & a \end{pmatrix}$$

### Problem 3.

Find all stationary points of  $f$ , classify them as local maximum/minimum points or saddle points, and determine whether  $f$  has global maximum/minimum values:

- a)  $f(x,y,z) = xy + xz - yz$     b)  $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$   
 c)  $f(x,y,z) = x^4 + y^4 + z^4 + z^2$     d)  $f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$

### Problem 4.

Determine whether  $f$  is a convex or concave function:

- a)  $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$     b)  $f(x,y,z) = e^{x-2y+z}$   
 c)  $f(x,y,z) = x^4 + y^4 + z^4 + z^2$     d)  $f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$   
 e)  $f(x,y,z) = \frac{xy + xz + yz}{xyz}$  defined for  $x,y,z > 0$

## Problems from the Workbook

Workbook [W]	6.1 - 6.26 (full solutions in the workbook)	
Exam problems	Midterm exam 10/2017 Question 6-8,	Midterm exam 01/2018 Question 7
	Midterm exam 01/2019 Question 1-8,	Midterm exam 05/2018 Question 7

## Answers to Key Problems

### Problem 1.

- a) Positive semi-definite    b) Positive semi-definite    c) Indefinite    d) Positive definite

### Problem 2.

It is negative semi-definite for  $a \leq -1$ .

### Problem 3.

- a) Saddle point  $(0,0,0)$ , no global max/min value  
b) Local min  $(0,0,0,0)$ , global min value  $f_{\min} = 0$ , no global max value  
c) Local min  $(0,0,0)$ , global min value  $f_{\min} = 0$ , no global max value  
d) Local max  $(0,0,0)$ , global max value  $f_{\max} = 16$ , no global min value

### Problem 4.

- a) convex    b) convex    c) convex    d) concave    e) convex