Solutions: Problem Set 4

1.

a) $A = \begin{pmatrix} 3 & 7 \\ 7 & 3 \end{pmatrix}$ $\quad \begin{vmatrix} 3-\lambda & 7 \\ 7 & 3-\lambda \end{vmatrix} = 0$

$$\lambda^2 - 6\lambda - 40 = 0$$

$$\underline{\lambda_1 = -4}, \ \underline{\lambda_2 = 10}$$

$\underline{E_{-4}}$: $\begin{pmatrix} 7 & 7 \\ 7 & 7 \end{pmatrix} \to \begin{pmatrix} 7 & 7 \\ 0 & 0 \end{pmatrix}$ $\quad 7x + 7y = 0$, y free

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix} = y \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\quad \underline{u_1} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Base of E_{-4}: $\{\underline{u_1}\}$

$\underline{E_{10}}$: $\begin{pmatrix} -7 & 7 \\ 7 & -7 \end{pmatrix} \to \begin{pmatrix} -7 & 7 \\ 0 & 0 \end{pmatrix}$ $\quad -7x + 7y = 0$, y free

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ y \end{pmatrix} = y \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad \underline{u_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Base of E_{10}: $\{\underline{u_2}\}$

b) $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$ $\quad \begin{vmatrix} 1-\lambda & 1 \\ -1 & 3-\lambda \end{vmatrix} = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$\underline{\lambda_1 = \lambda_2 = 2}$

$\underline{E_2}$: $\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \to \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}$ $\quad -x+y=0$

y free

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ y \end{pmatrix} = y \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad \underline{v_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Base of E_2: $B = \{\underline{v_1}\}$

c) $A = \begin{pmatrix} 2 & -4 \\ 3 & -1 \end{pmatrix}$ $\quad \begin{vmatrix} 2-\lambda & -4 \\ 3 & -1-\lambda \end{vmatrix} = 0$

$\lambda^2 - \lambda + 10 = 0$

$\lambda = \dfrac{1 \pm \sqrt{1 - 4 \cdot 10}}{2}$

no eigenvalues

(among real number)

$\Rightarrow$ no eigenvectors

d) $A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 4 \end{pmatrix}$ $\quad \begin{vmatrix} 4-\lambda & 0 & 1 \\ 0 & 5-\lambda & 0 \\ 1 & 0 & 4-\lambda \end{vmatrix} = 0$

$(5-\lambda)(\lambda^2 - 8\lambda + 15) = 0$

$\lambda_1 = 5 \quad \lambda_2 = 5, \ \lambda_3 = 3$

E_5: $\begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix}$ $\quad$ $x = z$, y free, z free $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\underline{v_1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ \underline{v_2} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $\quad$ Base for E_5: $\{\underline{v_1}, \underline{v_2}\}$

E_3: $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ $\quad$ $x = -z$, $2y = 0$, z free $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -z \\ 0 \\ z \end{pmatrix} = z \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$\underline{v_3} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $\quad$ Base for E_3: $\{\underline{v_3}\}$

e) $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ $\quad \begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$

$(2-\lambda)(\lambda^2 - 4\lambda + 3) - 1\cdot(2-\lambda-1) + 1\cdot(1-(2-\lambda)) = 0$

$(2-\lambda)(\lambda-3)(\lambda-1) + (\lambda-1) + (\lambda-1) = 0$

$(\lambda-1)[(2-\lambda)(\lambda-3) + 2] = 0$

$\lambda_1 = 1$ or $-\lambda^2 + 5\lambda - 4 = 0$

$\lambda_2 = 1, \ \lambda_3 = 4$

$\underline{E_1}$: $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $\quad x+y+z=0$, y, z free $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y-z \\ y \\ z \end{pmatrix}$

$$= y \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

Base for E_1: $\left\{ \underline{u_1} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \underline{u_2} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$

$\underline{E_4}$: $\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{pmatrix}$ $\quad x = -y + 2z = z$, $y = z$, z free

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ z \\ z \end{pmatrix} = z \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad$ Base for E_4: $\left\{ \underline{u_3} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

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f) $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ $\quad \underline{\lambda_1 = \lambda_2 = \lambda_3 = 2}$

Since A is upper triangular

$\underline{E_2}$: $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad y=0$, $z=0$, x free $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} = x \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Base for E_2: $\left\{ \underline{u_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$

2.

a) A is diagonalizable with

$D = \begin{pmatrix} -4 & 0 \\ 0 & 10 \end{pmatrix}$, $P = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$

b) A is not diagonalizable

($\dim E_2 = 1 < 2 \leftarrow$ multiplicity of $\lambda = 2$)

c) A is not diagonalizable

(not enough eigenvalues)

d) A is diagonalizable with

$D = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, $P = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

e) A is diagonalizable with

$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$, $P = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

f) A is <u>not</u> diagonalizable

$\left(\dim E_2 = 1 < 3 \Leftarrow \text{mult. of } \lambda = 2 \right)$

<u>3.</u>

$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

A symmetric $\Downarrow$ A <u>diagonalizable</u>

$$\begin{vmatrix} 1-\lambda & 0 & 0 & -1 \\ 0 & 1-\lambda & -1 & 0 \\ 0 & -1 & 1-\lambda & 0 \\ -1 & 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda) \cdot \left[(1-\lambda) \cdot (\lambda^2 - 2\lambda) \right] - (-1) \left[(-1)(\lambda^2 - 2\lambda) \right] = 0$$

$$(1-\lambda)^2 (\lambda^2 - 2\lambda) - 1 \cdot (\lambda^2 - 2\lambda) = 0$$

$$(\lambda^2 - 2\lambda) \cdot \left((1-\lambda)^2 - 1 \right) = 0$$

$$\lambda(\lambda - 2) \cdot (\lambda^2 - 2\lambda) = 0$$

$$\lambda(\lambda - 2) \cdot \lambda \cdot (\lambda - 2) = 0$$

$$\underline{\underline{\lambda_1 = \lambda_2 = 0, \quad \lambda_3 = \lambda_4 = 2}}$$

4.

a) $A = \begin{pmatrix} 0.30 & 0.15 \\ 0.70 & 0.85 \end{pmatrix}$

$tr(A) = 0.30 + 0.85 = 1.15$

$|A| = 0.30 \cdot 0.85 - 0.70 \cdot 0.15 = 0.15$

Eigenvalues:

$$\left.\begin{aligned} &\begin{vmatrix} 0.3-\lambda & 0.15 \\ 0.7 & 0.85-\lambda \end{vmatrix} = 0 \\ &\lambda^2 - tr(A)\lambda + |A| = 0 \\ &\lambda^2 - 1.15\lambda + 0.15 = 0 \\ &\underline{\lambda_1 = 1}, \quad \lambda_2 = \underline{0.15} \end{aligned}\right\} \Rightarrow D = \begin{pmatrix} 1 & 0 \\ 0 & 0.15 \end{pmatrix}$$

Eigenvectors:

$\underline{E_1}$: $\left(\begin{array}{cc|c} -0.70 & 0.15 & 0 \\ \cancel{0.70} & \cancel{-0.15} & \cancel{0} \end{array}\right)$

one free var

$\underline{v} = \begin{pmatrix} x \\ y \end{pmatrix} = \underline{\begin{pmatrix} 15 \\ 70 \end{pmatrix} s} \Rightarrow \underline{v_1} = \begin{pmatrix} 15 \\ 70 \end{pmatrix}$

$\underline{E_{0.15}}$: $\left(\begin{array}{cc|c} 0.15 & 0.15 & 0 \\ \cancel{0.70} & \cancel{0.70} & \cancel{0} \end{array}\right)$

one free var y

$x = -y$

$\underline{v} = \begin{pmatrix} -y \\ y \end{pmatrix} = y \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} \Rightarrow \underline{v_2} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$P = \begin{pmatrix} 15 & -1 \\ 70 & 1 \end{pmatrix}$

$P^{-1} = \frac{1}{85}\begin{pmatrix} 1 & 1 \\ -70 & 15 \end{pmatrix}$

$\underline{\text{Compute } A^m}$ for m large $(m \to \infty)$

$P^{-1}AP = D \Rightarrow A = PDP^{-1}$

$\Rightarrow A^m = (PD\cancel{P^{-1}})(\cancel{P}D\cancel{P^{-1}})\cdots(\cancel{P}DP^{-1})$

$= PD^mP^{-1}$

$= \begin{pmatrix} 15 & -1 \\ 70 & 1 \end{pmatrix} \begin{pmatrix} 1^m & 0 \\ 0 & 0.15^m \end{pmatrix} \frac{1}{85} \begin{pmatrix} 1 & 1 \\ -70 & 15 \end{pmatrix}$

$\to \begin{pmatrix} 15 & -1 \\ 70 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \frac{1}{85} \begin{pmatrix} 1 & 1 \\ -70 & 15 \end{pmatrix}$

$= \begin{pmatrix} 15 & 0 \\ 70 & 0 \end{pmatrix} \cdot \frac{1}{85} \begin{pmatrix} 1 & 1 \\ -70 & 15 \end{pmatrix}$

$= \underline{\underline{\frac{1}{85} \begin{pmatrix} 15 & 15 \\ 70 & 70 \end{pmatrix}}} = (\underline{v} \mid \underline{v}), \quad \underline{v} = \begin{pmatrix} 15/85 \\ 70/85 \end{pmatrix}$

$\underline{\text{Equilibrium state:}}$

$\underline{V_0} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \Rightarrow \underline{V_m} = A^m \underline{v_0} \to (\underline{v} \mid \underline{v}) \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$

initial state

$x_0 + y_0 = 1$

$= x_0 \underline{v} + y_0 \underline{v}$

$= (x_0 + y_0)\underline{v}$

$= 1 \cdot \underline{v} = \underline{v}$

$\underline{\text{Eq. state:}} \quad \underline{v} = \underline{\underline{\begin{pmatrix} 15/85 \\ 70/85 \end{pmatrix}}}$

$\simeq \begin{pmatrix} 0.176 \\ 0.824 \end{pmatrix}$

b) $A = \begin{pmatrix} 0.86 & 0.42 \\ 0.14 & 0.58 \end{pmatrix}$ $\quad tr(A) = 1.44$, $|A| = 0.44$

Eigenvalues:

$$\lambda^2 - 1.44\lambda + 0.44 = 0$$

$$\underline{\lambda_1 = 1}, \quad \lambda_2 = \underline{0.44} \qquad D = \begin{pmatrix} 1 & 0 \\ 0 & 0.44 \end{pmatrix}$$

Eigenvectors:

E_1: $\begin{pmatrix} -0.14 & 0.42 \\ \cancel{0.14} & \cancel{-0.42} \end{pmatrix}$

$$-0.14x + 0.42y = 0$$

$$x = \underline{3y}$$

$$\underline{v} = \begin{pmatrix} 3y \\ y \end{pmatrix} = y\begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad \underline{v_1} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$E_{0.44}$: $\begin{pmatrix} 0.42 & 0.42 \\ \cancel{0.14} & \cancel{0.14} \end{pmatrix}$

$$x + y = 0 \Rightarrow x = -y$$

$$\underline{v} = \begin{pmatrix} -y \\ y \end{pmatrix} = y\begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \underline{V_2} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$$

$$P^{-1} = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$$

$$A^n = (PDP^{-1}) \ldots (PDP^{-1}) = PD^n P^{-1}$$

$$\rightarrow \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\frac{1}{4}\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 3 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3/4 & 3/4 \\ 1/4 & 1/4 \end{pmatrix}$$

Eq. state: $\underline{v_n} \rightarrow \begin{pmatrix} 3/4 & 3/4 \\ 1/4 & 1/4 \end{pmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}}}$

c)

$$A = \begin{pmatrix} 0.75 & 0.02 & 0.10 \\ 0.20 & 0.90 & 0.20 \\ 0.05 & 0.08 & 0.70 \end{pmatrix}$$

Use Wolfram Alpha to compute eigenvalues / eigenvectors:

$\underline{\lambda_1 = 1}$, $\lambda_2 = \underline{0.70}$, $\lambda_3 = \underline{0.65}$

$$\underline{E_1}: \underbrace{\underline{v_1} = \begin{pmatrix} 2 \\ 10 \\ 3 \end{pmatrix} \quad \underline{E_{0.35}}: \underline{v_2} = \begin{pmatrix} -8 \\ 5 \\ 3 \end{pmatrix} \quad E_{0.65}: \underline{v_3} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}_{\text{bases for eigenspaces}}$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.65 \end{pmatrix} \qquad P = \begin{pmatrix} 2 & -8 & -1 \\ 10 & 5 & 0 \\ 3 & 3 & 1 \end{pmatrix}$$

$$|P| = -1 \cdot 15 + 1 \cdot 90 = \underline{75} \qquad P^{-1} = \frac{1}{75} \begin{pmatrix} 5 & 5 & 5 \\ -10 & 5 & -10 \\ 15 & -30 & 90 \end{pmatrix}$$

$$A^n = P D^n P^{-1} \Rightarrow \begin{pmatrix} 2 & -8 & -1 \\ 10 & 5 & 0 \\ 3 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{1}{75} \begin{pmatrix} 5 & 5 & 5 \\ -10 & 5 & -10 \\ 15 & -30 & 90 \end{pmatrix}$$

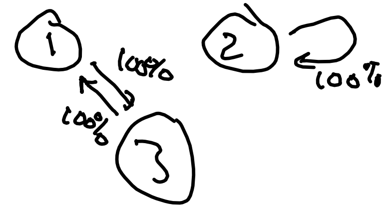
$$= \begin{pmatrix} 2 & 0 & 0 \\ 10 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix} \frac{1}{75} \begin{pmatrix} 5 & 5 & 5 \\ -10 & 5 & -10 \\ 15 & -30 & 90 \end{pmatrix} = \frac{1}{75} \begin{pmatrix} 10 & 10 & 10 \\ 50 & 50 & 50 \\ 15 & 15 & 15 \end{pmatrix}$$

$$= \frac{1}{15} \begin{pmatrix} 2 & 2 & 2 \\ 10 & 10 & 10 \\ 3 & 3 & 3 \end{pmatrix}$$

$$\underline{\text{Eq. state}}: \underline{v_n} \to \frac{1}{15} \begin{pmatrix} 2 & 2 & 2 \\ 10 & 10 & 10 \\ 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} 2/15 \\ 10/15 \\ 3/15 \end{pmatrix}$$

$v_0 = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$, $x_0 + y_0 + z_0 = 1$

d) $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$



In this case, it is possible to see what the result will be without eigenvalues / eigenvectors:

$$\underline{V_0} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \to \underline{V_1} = \begin{pmatrix} z_0 \\ y_0 \\ x_0 \end{pmatrix} \to \underline{V_2} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \to \dots$$

$$A^m = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad m \text{ odd}$$

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad n \text{ even}$$

$\Downarrow$

<u>no convergence</u> for A^m.

$x_0 = z_0$ } Eq. state is $\underline{\underline{\begin{pmatrix} x_0 \\ y_0 \\ x_0 \end{pmatrix}}}$

all other cases } no eq. state (no convergence)

e) $A = \begin{pmatrix} 0.2 & 0.4 & 0 \\ 0.8 & 0.4 & 0.7 \\ 0 & 0.2 & 0.3 \end{pmatrix}$

Use Wolfram Alpha:

$\lambda_1 = 1, \quad \lambda_2, \lambda_3 = \frac{1}{20}(-1 \pm \sqrt{41})$

$\simeq -0.37, \; 0.27$

$\underline{E_1}$: $\underline{v} = \begin{pmatrix} 7 \\ 14 \\ 4 \end{pmatrix} s \quad \underline{v_1} = \begin{pmatrix} 7 \\ 14 \\ 4 \end{pmatrix}$

$$A^m \to \underset{P}{\begin{pmatrix} 7 & \cdot & \cdot \\ 14 & \cdot & \cdot \\ 4 & \cdot & \cdot \end{pmatrix}} \underset{D^m}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}} \underset{P^{-1}}{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}}$$

$$= \begin{pmatrix} 7 & 0 & 0 \\ 14 & 0 & 0 \\ 4 & 0 & 0 \end{pmatrix} P^{-1} = \underline{\begin{pmatrix} 7/25 & 7/25 & 7/25 \\ 14/25 & 14/25 & 14/25 \\ 4/25 & 4/25 & 4/25 \end{pmatrix}}$$

$\underline{\text{Eq. state}}$: $\underline{v_m} \to \begin{pmatrix} 7/25 \\ 14/25 \\ 4/25 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0.28 \\ 0.56 \\ 0.16 \end{pmatrix}}}$

(In Wolfram Alpha, the command

[[0.2, 0.4, 0], [0.8, 0.4, 0.7], [0, 0.2, 0.3]]

gives $S \leftarrow P = (\underline{v_1} | \underline{v_2} | \underline{v_3})$

$J \leftarrow D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$)