Key Problems

Problem 1.
Use Gaussian elimination to solve the linear systems with the following augmented matrices:

a) $\left(\begin{array}{rrr|r} 1 & 3 & 4 & 11 \\ 2 & -1 & 3 & 3 \\ 3 & 2 & 5 & 12 \end{array}\right)$ b) $\left(\begin{array}{rrr|r} 1 & 3 & 4 & 11 \\ 2 & -1 & 3 & 3 \\ 3 & 2 & 7 & 12 \end{array}\right)$ c) $\left(\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & 8 \\ 1 & 3 & 1 & 5 & 28 \\ 2 & 4 & 2 & 9 & 48 \end{array}\right)$

Problem 2.
Determine how many solutions the linear system has:

$$\begin{array}{rcrcrcr} x & + & y & + & 2z & = & 6 \\ x & + & 2y & + & 4z & = & 13 \\ x & + & 3y & + & 9z & = & 24 \end{array}$$

Does the number of solutions change if we change the blue coefficient in the first equation? In that case, determine how the number of solutions changes with the blue coefficient.

Problem 3.
We consider the homogeneous linear system with coefficient matrix

$$A = \begin{pmatrix} 1 & 1 & 4 & -1 \\ 5 & 5 & -1 & 4 \\ 7 & 6 & 3 & 3 \end{pmatrix}$$

Describe the set of solutions geometrically. How many degrees of freedom are there? Does this change if we change the red coefficient in the second row?

Problems from the Workbook and Lecture Notes

Exercise problems:	Eriksen [E] 1.1 - 1.16 (see It's Learning)
Optional problems:	Workbook [W] 1.1 - 1.18 (some problems are the same as the ones in [E])

Answers to Key Problems

Problem 1.
a) $(x,y,z) = (1,2,1)$ b) No solutions c) $(x,y,z,w) = (2-z, 2, z, 4)$ with z free

Problem 2.
There is one unique solution. The number of solutions only changes if the blue coefficient is -1, in which case there are no solutions. For any other value, there is a unique solution.

Problem 3.
We have that $\text{rk}(A) = 3$, and there is $n - \text{rk}(A) = 4 - 3 = 1$ degrees of freedom. Therefore the set of solutions is a straight line in $\mathbb{R}^4$. If we change the red coefficient, the rank of A remains $\text{rk}(A) = 3$ unless the coefficient is 6, in which case $\text{rk}(A) = 2$. Therefore, the set of solutions is a line for all values of the red coefficient except 6, and in this case the set of solutions is a plane since the dimension is $n - \text{rk}(A) = 4 - 2 = 2$.

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