EXAMINATION QUESTION PAPER - Course paper

EBA 29103
Mathematics for Business Analytics

Department of Economics

Start date:	19.10.2020	Time 09.00
Finish date:	26.10.2020	Time 12.00

Weight:	Pass / Fail
Total no. of pages:	3 incl. front page
No. of attachments files to question paper:	0
To be answered:	Individually

Answer paper size:	No limit. excl. attachments
Max no. of answer paper attachment files:	0
Allowed answer paper file types:	pdf
Re-sit	Ordinary

Course paper	EBA 29103 Mathematics for Business Analytics
Deadline	October 26th 2020 at 1200

The problem set consists of two pages. All 24 subquestion have equal weight, and at least 60% score is required to pass. **You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.**

Your answers should be provided as a single file in PDF format. You are encouraged to write with a pen and scan your paper. Check that the resulting file is easy to read. For more information, see `https://portal.bi.no/en/examination/digital-examination/digital-submission/`.

Question 1.
Compute the indefinite integrals:

a) $\displaystyle\int \frac{1}{x\sqrt{x}}\,\mathrm{d}x$ b) $\displaystyle\int \frac{6-2x}{x^3}\,\mathrm{d}x$ c) $\displaystyle\int \frac{x^2}{1+x}\,\mathrm{d}x$ d) $\displaystyle\int 16(4+2x)^7\,\mathrm{d}x$

Question 2.
Use Gaussian elimination to solve the linear systems. Show elementary row operations, mark the pivot positions in the echelon form, and specify the number of solutions.

$$a)\quad \begin{array}{rcrcrcr} x & - & 2y & + & 3z & = & 6 \\ 2x & & & - & z & = & 1 \\ x & - & 2y & + & 5z & = & 8 \end{array} \qquad b)\quad \begin{array}{rcrcrcr} 3x & + & 6y & + & 12z & = & 15 \\ -3x & + & y & + & 5z & = & -5 \\ 5x & + & 3y & + & 3z & = & 15 \end{array}$$

Question 3.
We consider the function given by

$$f(x) = \frac{1-\sqrt{x}}{x+3}$$

a) Compute $f'(x)$, and determine when f is increasing and decreasing.

b) Find the maximum and minimum value of f, if they exist.

Question 4.
We consider the matrices A, E_1, E_2 og E_3 given by

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 10 & 8 \\ -3 & 2 & 4 \end{pmatrix}, \quad E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}, \quad E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

Compute the following expressions:

a) A^{-1} b) $A^T \cdot A$ c) $E_3 \cdot E_2 \cdot E_1 \cdot A$

Question 5.
Compute the integrals:

a) $\displaystyle\int_0^1 x|x|\,\mathrm{d}x$ b) $\displaystyle\int_0^1 \frac{1}{x^2}\,\mathrm{d}x$ c) $\displaystyle\int_0^\infty x\,e^{-x^2}\,\mathrm{d}x$

Question 6.
A linear system $A\mathbf{x} = \mathbf{b}$ has coefficient matrix A given below. Compute the determinant $|A|$, and determine when the linear system has exactly one solution:

a) $A = \begin{pmatrix} 1 & 1 & s \\ 2 & s & 4 \\ 1 & 1 & 3 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & t & t \\ t & 1 & t \end{pmatrix}$ c) $A = \begin{pmatrix} t & 1 & t \\ 1 & t & -1 \\ t & -1 & t \end{pmatrix}$

Question 7.
Compute the indefinite integrals:

a) $\displaystyle\int 5x\sqrt{x}\,\ln(x)\,\mathrm{d}x$ b) $\displaystyle\int \frac{x}{e^x}\,\mathrm{d}x$ c) $\displaystyle\int \frac{2x+2}{4-x^2}\,\mathrm{d}x$ d) $\displaystyle\int \frac{\sqrt{x}}{1-\sqrt{x}}\,\mathrm{d}x$

Question 8.
We consider the linear system $A\mathbf{x} = \mathbf{b}$ when the matrix A and the vectors $\mathbf{x}$ and $\mathbf{b}$ are given by

$$A = \begin{pmatrix} t & 1 & 1 \\ 2 & 1 & t \\ 4 & t & 2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ t \\ 4 \end{pmatrix}$$

Determine how many solutions the linear system has for all values of the parameter t, and find all solutions when the system is consistent. Use Kramer's rule when the system has a unique solution.

Question 9.
We consider the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ and the matrix A with these four vectors as columns:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 0 \\ 4 \\ 3 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 1 \\ 12 \\ 5 \\ 16 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 3 & 2 & 1 \\ 2 & -1 & 0 & 12 \\ 1 & 2 & 4 & 5 \\ 3 & 1 & 3 & 16 \end{pmatrix}$$

a) Determine whether any of the vectors can be written as a linear combination of the other vectors. If this is the case, write down one such expression.

b) Compute $\det(A^T A)$.