EBA2911 Mathematics for Business Analytics
autumn 2021
Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

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Lecture 9
on Monday 11 Oct. 15.00-16.45
Sec. 4.7, 7.8-10
Rational functions and asymptotes. Continuity and the intermediate value theorem.

Here are recommended exercises from the textbook [SHSC].

Section **4.7** exercise 4
Section **7.9** exercise 1-5
Section **7.8** exercise 1-5
Section **7.10** exercise 1-2

Problems for the exercise session Wednesday 13 Oct.

Problem 1 Determine the expression $f(x) = c + \frac{a}{x-b}$ of the hyperbolas (a-d) in figure 1.

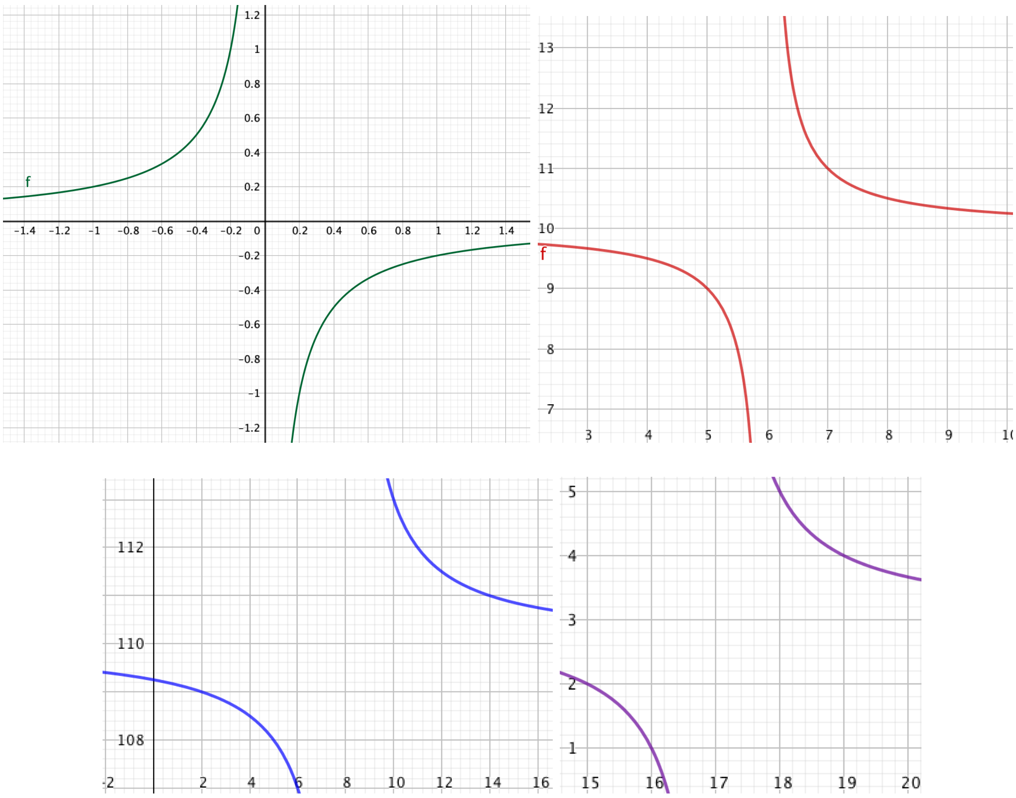

Figure 1: Hyperbolas a-d

Problem 2 Determine the asymptotes of the hyperbolas (a-d) in Problem 8.

Problem 3 Determine the asymptotes of the rational functions.

a) $f(x)=\frac{4x-10}{x-3}$ b) $f(x)=\frac{70-40x}{3-2x}$ c) $f(x)=\frac{12}{x^2+3}$

d) $f(x)=\frac{4x^2-28x+40}{x^2-4x+3}$ e) $f(x)=\frac{x^2+3x+5}{x-7}$ f) $f(x)=\frac{x^3-8}{x^2-10x+16}$

Problem 4 (Multiple choice exam spring 2018, Problem 8, translated) The function

$$f(x)=\frac{2x^2+5x-7}{x^2-2x+3}$$

Which statement is true?
A) The function has only vertical asymptotes.
B) The function has only horizontal asymptotes.
C) The function has one vertical and one horizontal asymptote.
D) The function has two vertical and one horizontal asymptote.
E) I choose not to answer this question.

Problem 5 Determine if the function $f(x)$ has a zero in the interval I. Hint: The intermediate value theorem!
a) $f(x)=\sqrt{x-2}-x+3$ and $I=[4,5]$
b) $f(x)=(x-5)\sqrt{(0.2x+5)}-0.2(x-3)^2$ and $I=[5,15]$
c) $f(x)=\frac{4x-10}{x-3}-4$ and $I=[2,4]$

Answers

Problem 1
a) $f(x)=-\frac{1}{5x}$ b) $f(x)=10+\frac{1}{x-6}$ c) $f(x)=110+\frac{6}{x-8}$ d) $f(x)=3+\frac{2}{x-17}$

Problem 2
a) vertical asymptote: $x=0$, horizontal asymptote: $y=0$
b) vertical asymptote: $x=6$, horizontal asymptote: $y=10$
c) vertical asymptote: $x=8$, horizontal asymptote: $y=110$
d) vertical asymptote: $x=17$, horizontal asymptote: $y=3$

Problem 3
a) $f(x)=4+\frac{2}{x-3}$ so vertical asymptote: $x=3$, horizontal asymptote: $y=4$
b) $f(x)=20-\frac{10}{2x-3}$ so vertical asymptote: $x=\frac{3}{2}$, horizontal asymptote: $y=20$
c) Since x^2+3 is positive for all x, $f(x)$ is defined for all x, so no vertical asymptote. Horizontal asymptote: $y=0$
d) $f(x)=4-\frac{4(3x-7)}{(x-1)(x-3)}$ so vertical asymptotes: $x=1$ and $x=3$, horizontal asymptote: $y=4$
e) $f(x)=x+10+\frac{75}{x-7}$ so vertical asymptote: $x=7$, non-vertical asymptote: $y=x+10$
f) $f(x)=x+10+\frac{84}{x-8}$ so vertical asymptote: $x=8$, non-vertical asymptote: $y=x+10$

Problem 4
Note that $x^2-2x+3=(x-1)^2+2$ which is never equal to 0. Hence there are no vertical asymptotes. This gives B.

We could also find the horizontal asymptote by polynomial division. We get

$$\begin{array}{l}\big(\;\;2x^2+5x\;\;-7\big):\big(x^2-2x+3\big)=2+\dfrac{9x-13}{x^2-2x+3}\\ \underline{\;-2x^2+4x\;\;-6\;}\\ \qquad\qquad 9x-13\end{array}$$

Since

$$\frac{9x-13}{x^2-2x+3}=\frac{\frac{9}{x}-\frac{13}{x^2}}{1-\frac{2}{x}+\frac{3}{x^2}}$$

approaches $\dfrac{0}{1} = 0$ when x (or $-x$) grows without bounds (i.e. $x \to \pm\infty$), it follows that

$$\frac{2x^2 + 5x - 7}{x^2 - 2x + 3}$$

approaches 2 when x (or $-x$) grows without bounds. So the horizontal line $y = 2$ (and x free) is a horizontal asymptote for $f(x)$.

Problem 5

a) $f(x)$ has a zero between $x = 4$ and $x = 5$ by the intermediate value theorem because $f(4) = \sqrt{4-2} - 4 + 3 = 0.41 > 0$ while $f(5) = \sqrt{5-2} - 5 + 3 = -0.27 < 0$ and the function is defined and is continuous on the whole interval.

b) $f(x)$ has a zero between $x = 5$ and $x = 6$ by the intermediate value theorem because $f(5) = -0.80$ while $f(6) = 0.69 > 0$ and the function is defined and is continuous on the whole interval.
Note: $f(15) = -0.52 < 0$ together with $f(6) > 0$ tell that $f(x)$ has a zero between $x = 6$ and $x = 15$. So f(x) has at least 2 zeros on the interval $[5, 15]$.

c) $f(x) = \frac{2}{x-3}$ har no zeros on the interval $I = [2, 4]$ because the equation $\frac{2}{x-3} = 0$ has no solutions. Note: We can not use the intermediate value theorem even if $f(2) = -2 < 0$ and $f(4) = 2 > 0$ because $f(x)$ is not defined on the whole interval (even if $f(x)$ er continuous for all x where it is defined).