

EBA2911 Mathematics for Business Analytics
autumn 2021
Exercises

I came to the position that mathematical analysis is not one of the many ways of doing economic theory: it is the only way.

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Lecture 15
on Wednesday 17 Nov. at 10-11.45
Sec. 7.12, 6.4, 9.3, 9.5:
l'Hôpital's rule. Marginal revenue and cost.

Here are recommended exercises from the textbook [SHSC].

Section 7.12 exercise 1-3, 4a, 5

Section 6.4 exercise 2, 6

Section 9.5 exercise 1-4

Problems for the exercise session Wednesday 17 Nov.
in B2-45 (at 12-15.45) or on Zoom

Problem 1 Compute the limit values.

a) $\lim_{x \rightarrow 3} \frac{-x}{25(x-1)}$	b) $\lim_{x \rightarrow \ln 5} \frac{e^x - 5}{x^2 - 5}$	c) $\lim_{x \rightarrow \ln 5} \frac{e^x - 5}{x^2 - (\ln 5)^2}$
d) $\lim_{x \rightarrow 0} \frac{7x}{e^x - 1}$	e) $\lim_{x \rightarrow 0} \frac{x^{10}}{e^x - 1}$	f) $\lim_{x \rightarrow 1} \frac{x \ln(x)}{x^2 - 1}$
g) $\lim_{x \rightarrow 1} \frac{\ln(x)}{e^{2x} - e^2}$	h) $\lim_{x \rightarrow 1} \frac{\ln(x)}{\sqrt{x} - 1}$	i) $\lim_{x \rightarrow 2} \frac{e^{x^2 - 3x + 2} - 1}{x^2 - 4}$

Problem 2 Compute the limit values by applying l'Hôpital's rule.

a) $\lim_{x \rightarrow \infty} \frac{-x}{25(x-1)}$	b) $\lim_{x \rightarrow 1} \frac{\ln(x)}{2x-2}$	c) $\lim_{x \rightarrow \infty} \frac{x^2 - 4x + 10}{e^x - 5}$	d) $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x}$
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Problem 3 Explain why $C(x)$ is a cost function by checking the three criteria:

(1) $C(0) > 0$

(2) $C(x)$ is an increasing function

(3) $C(x)$ is a convex function

Also determine the cost optimum and the average cost per unit at cost optimum (also called *the minimal unit cost* or *the optimal unit cost*).

a) $C(x) = 0.01x^2 + 8x + 2500, x \geq 0$

b) $C(x) = 0.05(x + 200)^2, x \geq 0$

c) $C(x) = 400e^{0.001x^2}, x \geq 0$

d) $C(x) = 50x + 1000, 0 \leq x \leq 1000$

Problem 4 $C(x)$ is the cost function, $R(x)$ is the revenue function and x is number of produced and sold units. Determine the profit maximising number of units.

a) $C(x) = 0.01x^2 + 8x + 2500$ and $R(x) = 100x$ for $x \geq 0$

b) $C(x) = 0.005x^2 + 20x + 30000$ and $R(x) = 50x$ for $0 \leq x \leq 2000$

Problem 5 I figure 1 you see the graph of four different cost functions.

- Order the cost functions from the one with the smallest minimal unit cost to the one with the largest minimal unit cost.
- Find an approximate value for the cost optimum for each of the cost functions.
- Find an approximate value for the minimal unit cost for each of the cost functions.

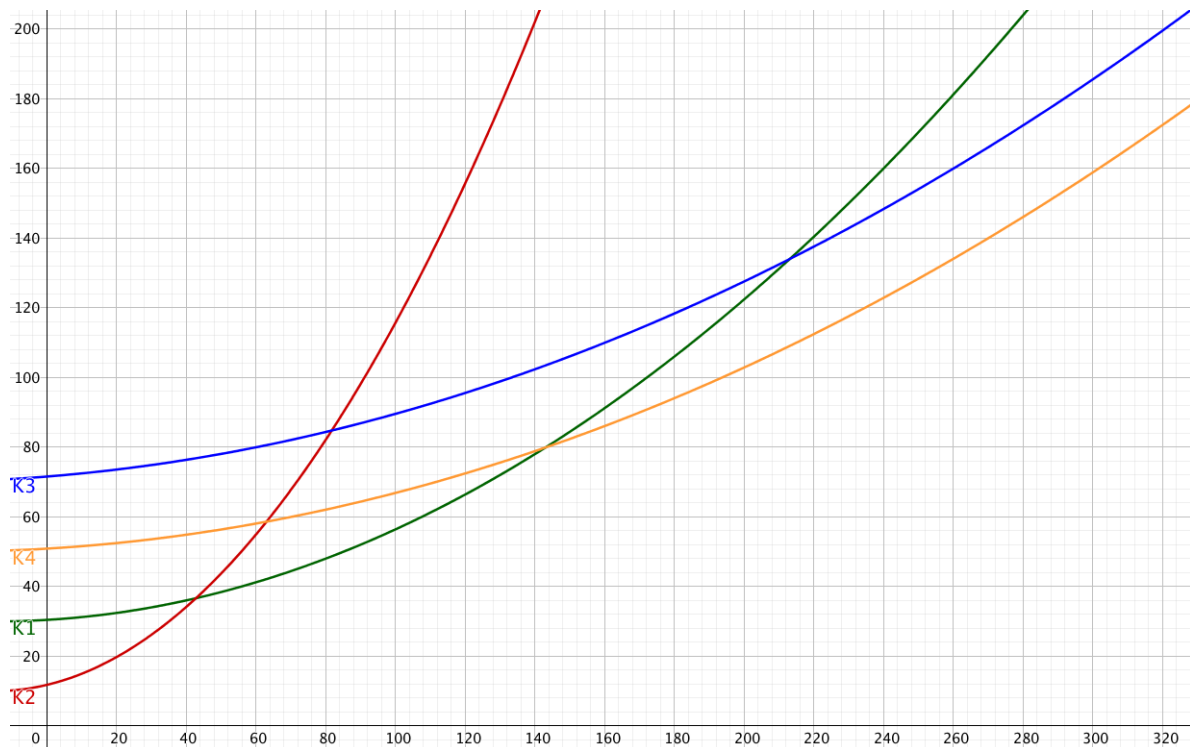


Figure 1: Four cost functions (K_1 – K_4)

Problem 6 (Multiple choice exam 2017s, problem 4)

A firm has the cost function $C(x) = 205x^3 - 120x^2 + 2000x + 2800$ when $x \geq 0$. What is the minimal average unit cost (the cost optimum)?

- 2 kr
- 12 kr
- 3980 kr
- 7960 kr
- I choose not to answer this problem.

Problem 7 (Multiple choice exam 2016a, problem 14)

We consider the limit value

$$\lim_{x \rightarrow \infty} \frac{1 - x \ln(x)}{e^x}$$

What is true?

- The limit value does not exist
- The limit value equals 1
- The limit value equals $-\frac{1}{2}$
- The limit value equals 0
- I choose not to answer this problem.

Problem 8 (Multiple choice exam 2015a, problem 15)

We consider the limit value

$$\lim_{x \rightarrow 1} \frac{\ln(x) - x + 1}{x^2 - 2x + 1}$$

What is true?

- (A) The limit value does not exist
- (B) The limit value equals 0
- (C) The limit value equals 1
- (D) The limit value equals $-\frac{1}{2}$
- (E) I choose not to answer this problem.

Answers

Problem 1

- a) $\frac{-3}{25(3-1)} = -0.06$ b) 0 c) $\frac{5}{2\ln 5}$
 d) 7 e) 0 f) 0.5
 g) $\frac{1}{2e^2}$ h) 2 i) $\frac{1}{4}$

Problem 2

- a) $\frac{-1}{25}$ b) $\frac{1}{2}$ c) $\lim_{x \rightarrow \infty} \frac{2}{e^x} = 0$ d) 0

Problem 3

- a) $C(0) = 2500 > 0$, $C'(x) = 0.02x + 8 > 0$ for $x > 0$ and so $C(x)$ is an increasing function for $x \geq 0$, $C''(x) = 0.02 > 0$ and so $C(x)$ is a convex function for $x \geq 0$. Cost optimum $x = 500$ gives minimal unit cost $A(500) = 18$
 b) $C(0) = 2000 > 0$, $C'(x) = 0.1x + 20 > 0$ for $x > 0$ and so $C(x)$ is a increasing function for $x \geq 0$, $C''(x) = 0.1 > 0$ and so $C(x)$ is a convex function for $x \geq 0$. Cost optimum $x = 200$ gives minimal unit cost $A(200) = 40$
 c) $C(0) = 400 > 0$, $C'(x) = 0.8xe^{0.001x^2} > 0$ for $x > 0$ and so $C(x)$ is an increasing function for $x \geq 0$, $C''(x) = 0.8(1 + 0.002x^2)e^{0.001x^2} > 0$ and so $C(x)$ is a convex function for $x \geq 0$. Cost optimum $x = 22.36$ gives minimal unit cost $A(22.36) = 29.49$
 d) $C(0) = 1000 > 0$, $C'(x) = 50 > 0$ and so $C(x)$ is an increasing function for $x \geq 0$, $C''(x) = 0 \geq 0$ and so $C(x)$ is a convex function for $x \geq 0$. Cost optimum $x = 1000$ gives minimal unit cost $A(1000) = 51$

Problem 4

- a) For $x = 4600$ the marginal cost equals the marginal revenue and $\pi''(x) = -0.02 < 0$ gives that the profit function is concave and hence $x = 4600$ is maximising the profit.
 b) For $x = 3000$ the marginal cost equals the marginal revenue, but this is outside the domain of definition for the modell. We see that $\pi'(x) = 30 - 0.01x$ is positive for $x < 3000$ which gives that the profit function is increasing for x in the interval $[0, 2000]$ and hence $x = 2000$ is maximising the profit.

Problem 5

- a) K_4, K_1, K_3, K_2
 b) $K_4 : x = 220$, $K_1 : x = 120$, $K_3 : x = 270$, $K_2 : x = 40$
 c) $A_4(220) = \frac{112}{220} = 0.51$, $A_1(120) = \frac{65}{120} = 0.54$, $A_3(270) = \frac{165}{270} = 0.61$, $A_2(40) = \frac{35}{40} = 0.88$

Problem 6 (Multiple choice exam 2017s, problem 4)

C

Problem 7 (Multiple choice exam 2016a, problem 14)

D

Problem 8 (Multiple choice exam 2015a, problem 15)

D