EBA 2911, lecture 8, 6 oct. 2021, Runar Ile

**Plan**
1. Increasing and decreasing functions
2. Circles and ellipses
3. Polynomial functions

## 1. Increasing and decreasing functions

**Ex** $f(x) = 0.03x^2 + 8x - 1500$, $D_f = [0, \rightarrow\rangle$

(meaning: $x \geq 0$)

Is $f(x)$ increasing?
—"— decreasing?
– or neither

Can look at the graph (by some tool)

or: Complete the square and draw the graph by hand

$$f(x) = 0.03\left[x^2 + \frac{800}{3}x\right] - 1500$$

$$= 0.03\left[\left(x + \frac{800}{6}\right)^2 - \left(\frac{800}{6}\right)^2\right] - 1500$$

$3 \cdot \cancel{160000}$

$$= 0.03\left(x + \frac{800}{6}\right)^2 - \cancel{0.03} \cdot \frac{\cancel{640000}}{\cancel{36}\ 9} - 1500$$

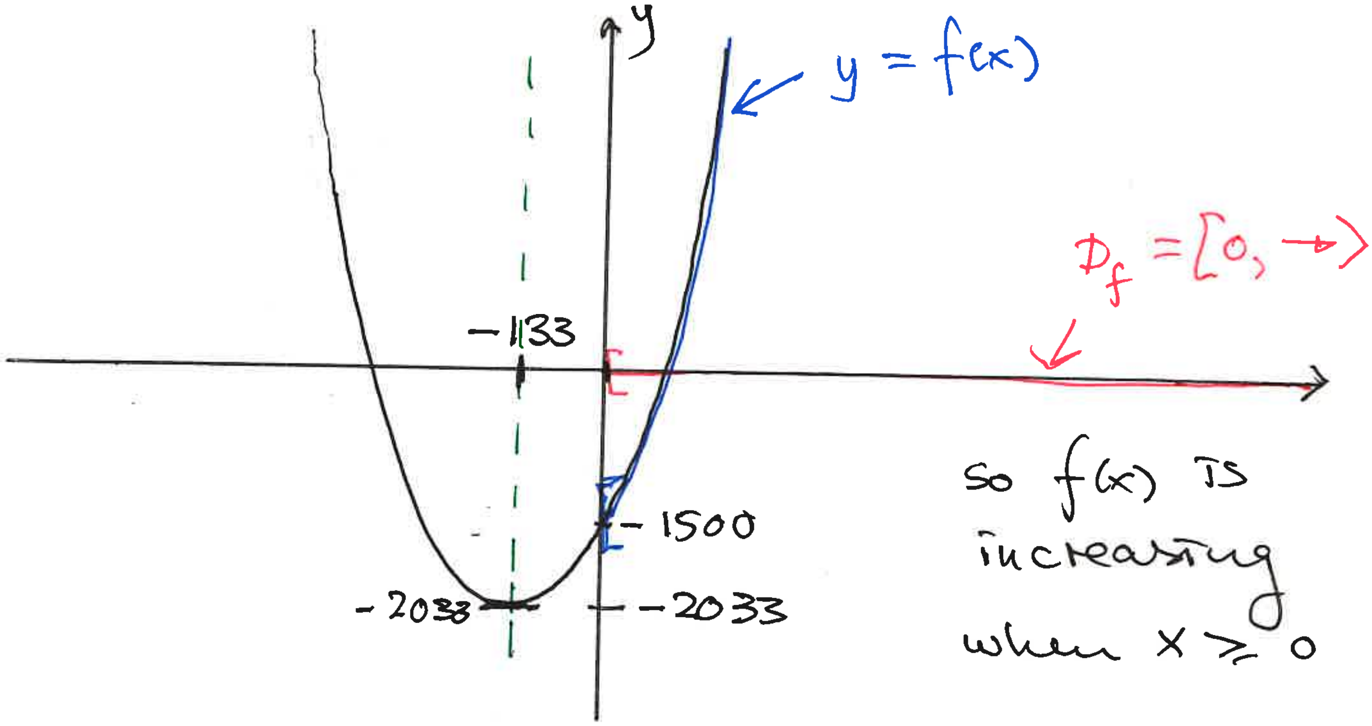
$$= 0.03\left(x + \frac{800}{6}\right)^2 - \frac{4800}{9} - \frac{13500}{9}$$

$$= \underbrace{0.03}_{a}\left(x + \frac{800}{6}\right)^2 - \frac{18300}{9}$$

Symmetry axis: $x = s = -\frac{800}{6} \approx -133$ (y free)

Minimum value: $y = f\left(-\frac{800}{6}\right) = -\frac{18300}{9} = \frac{6100}{3}$

$\approx -2033$




so $f(x)$ is increasing when $x \geq 0$

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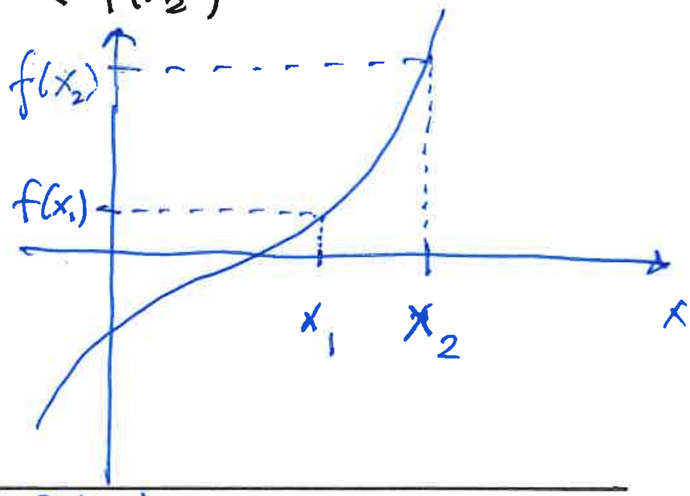
<u>Definition</u> A function $f(x)$ is <u>increasing</u> if for <u>all</u> $x_1 < x_2$

one has $\quad f(x_1) \leq f(x_2)$

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<u>Ex</u> $f(x) = 2x+5$ is strictly! increasing for all $x$ !

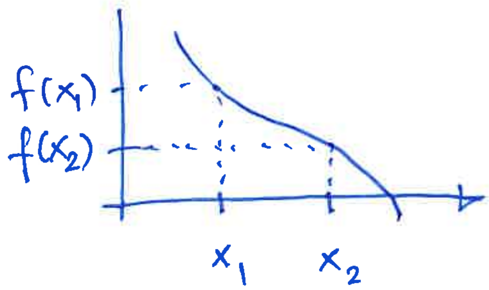
Reason: Assume $x_1 < x_2 \quad | \cdot 2$

$2x_1 < 2x_2 \quad | +5$

$$f(x_1) = 2x_1+5 < 2x_2+5 = f(x_2)$$

so $f(x)$ is (strictly) increasing

**Definition** A function $f(x)$ is <u>decreasing</u> if for all $x_1 < x_2$ one has $f(x_1) \geq f(x_2)$




**Problem** Show that $f(x) = -2x+5$ is (strictly) decreasing.

**Solution** Suppose $x_1 < x_2 \quad | \cdot (-2)$

$$-2x_1 > -2x_2 \quad | + 5$$

$$f(x_1) = -2x_1+5 > -2x_2+5 = f(x_2)$$

**Problem** We have the constant function $f(x)=5$. Decide whether $f(x)$ is increasing/decreasing/(neither)

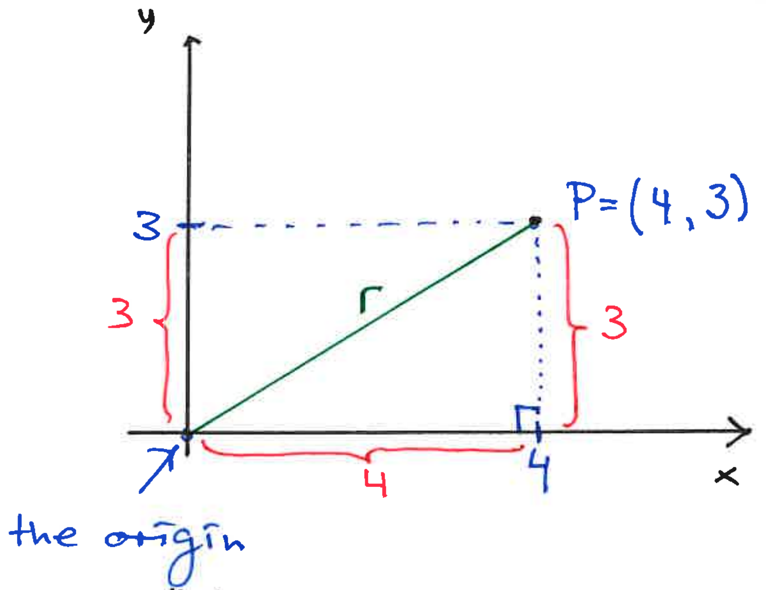
**Solution**

Increasing: If $x_1 < x_2$ then $f(x_1) = 5 \leq 5 = f(x_2)$

Decreasing: If $x_1 < x_2$ then $f(x_1) = 5 \geq 5 = f(x_2)$

– so both.

## 2. Circles and ellipses

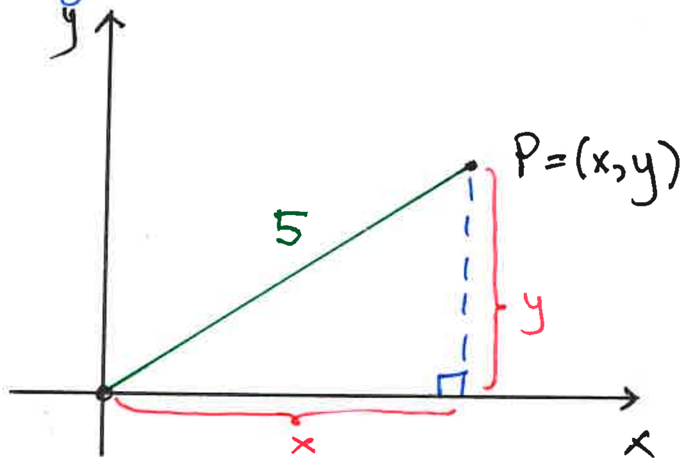



Pytagoras:

$r^2 = 4^2 + 3^2 \quad (r \geq 0)$

$r^2 = 16 + 9 = 25$

$\underline{r = 5} \; (= \sqrt{25})$

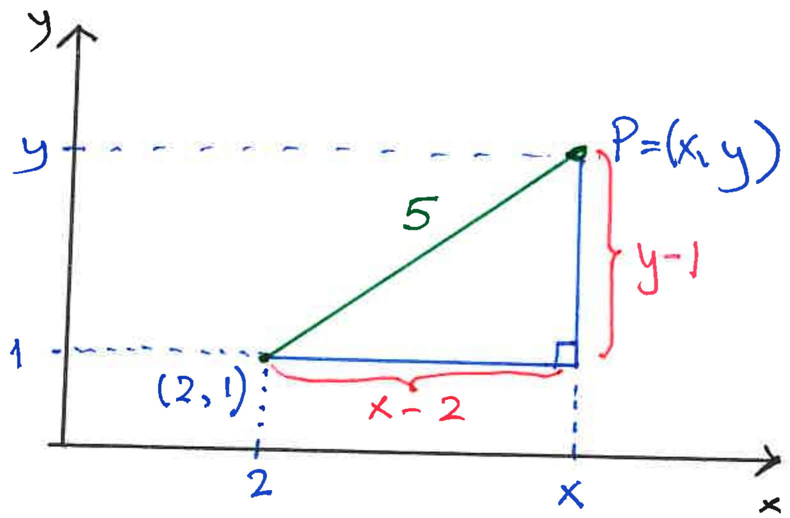



Pytagoras:

$25 = 5^2 = x^2 + y^2$

- one equation
- two unknowns
- infinitely many solutions

= the points on a circle with radius 5 and centre (0, 0).




Pytagoras:

$5^2 = (x-2)^2 + (y-1)^2$

$25 = x^2 - 4x + 4 + y^2 - 2y + 1$

that is:

$x^2 + y^2 - 4x - 2y = 20$

**Problem** Determine the radius and the centre of $x^2+y^2-2x+6y=-9$

**Solution**

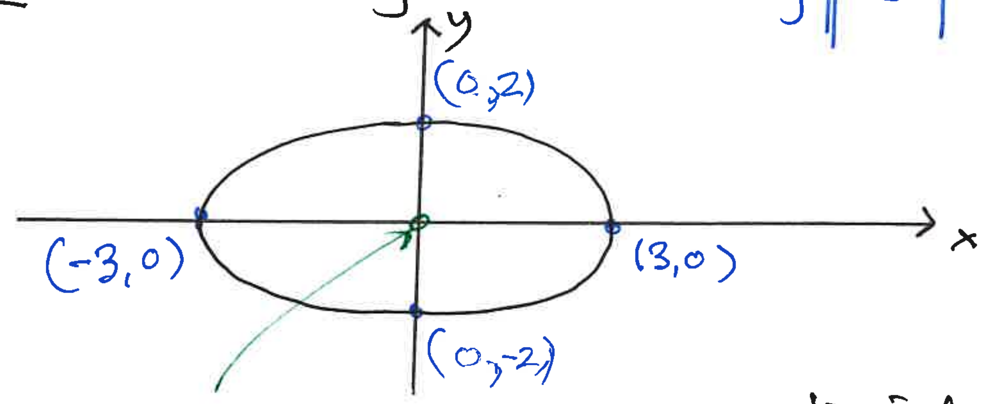
$$\underbrace{(x-1)^2}_{x^2-2x+1} + \underbrace{(y+3)^2}_{y^2-6y+9} = -9+1+9 = 1$$

Centre: $\underline{\underline{(1,-3)}}$, radius: $\sqrt{1} = \underline{\underline{1}}$

## Ellipses

**Ex** $4x^2+9y^2=36$

| $x$ | 3 | -3 | 0 | 0 |
|---|---|---|---|---|
| $y$ | 0 | 0 | 2 | -2 |




the centre of the ellipse

- divide each side by 36:

$$\frac{1}{9} = \frac{4}{36}\cdot x^2 + \underbrace{\frac{9}{36}}_{\frac{1}{4}}\cdot y^2 = 1$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

- remainds of a circle-equation

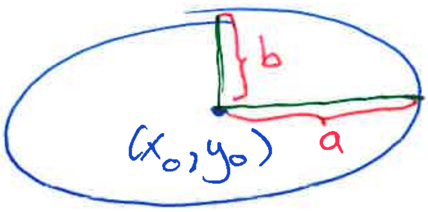
but the x-axis is streched by a factor 3

and the y-axis ——— " ——— 2

In general, any ellipse is the set of solutions of an equation of the form

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

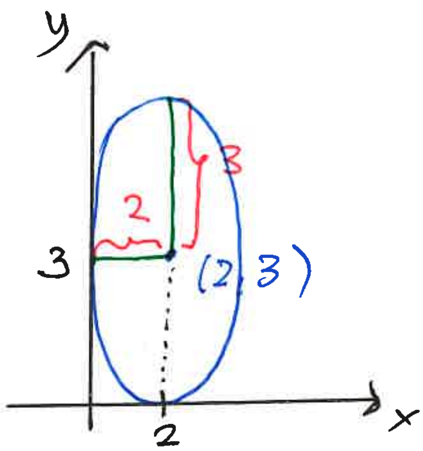
Here $(x_0, y_0)$ is the centre of the ellipse and $a$ and $b$ are the semi-axes




Ex $$\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$$

centre : $(2, 3)$

Semi-axes : $a = 2^{=\sqrt{4}}$ and $b = 3^{=\sqrt{9}}$



## 3. Polynomial functions

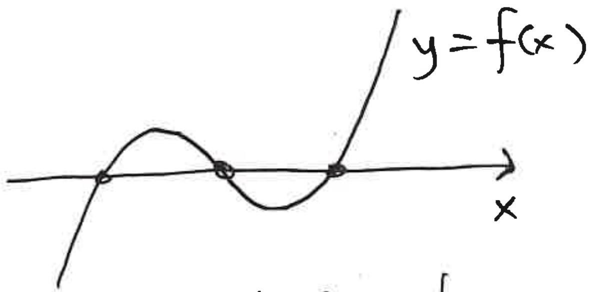
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

$a_n \neq 0$ then $\deg(f) = n$.

(degree)

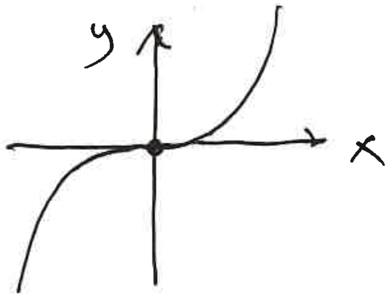
$f(x)$ has at most $n$ roots (zeros)

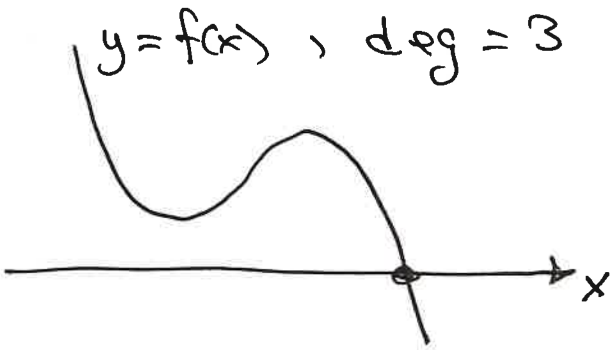
Ex



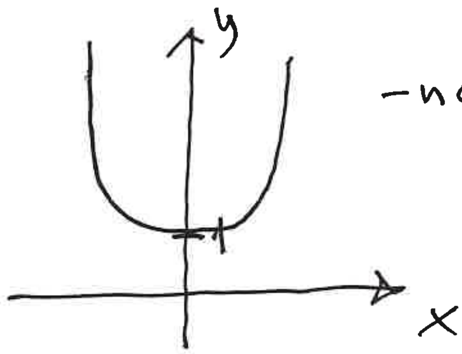

has degree 3

$y = x^3$:

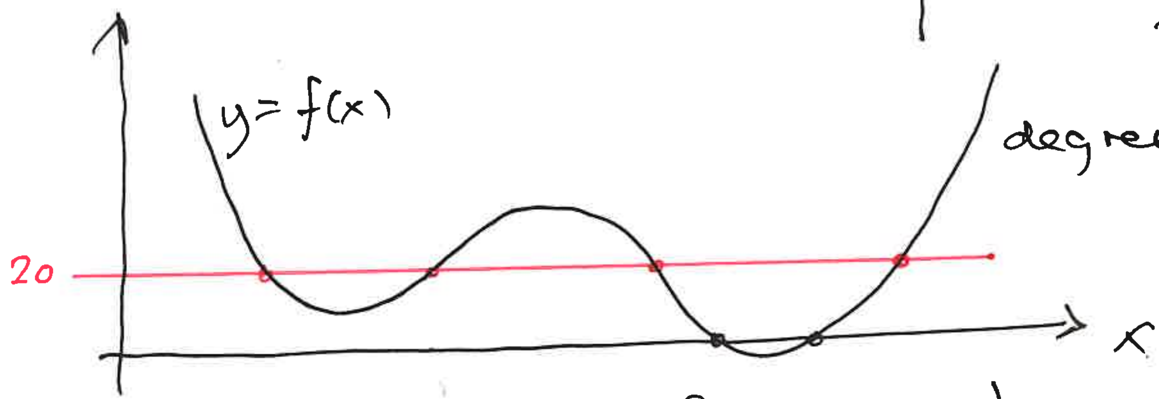



Ex. $f(x) = x^4 + 1$




– no roots!




degree 4

$f(x) = 20$ has four solutions (roots)

$$\underbrace{f(x) - 20}_{\text{still a polynomial of the same degree as } f(x).} = 0$$