EBA2911, lecture 6, 22 Sept 2021, Runar Ile

Plan
1. Polynomial division and factorisation
2. Rational and radical equations
3. Inequalities

1. Polynomial division and factorisation

Want to divide a polynomial $f(x)$ with a polynomial $g(x)$ and get a polynomial $q(x)$ with a remainder $r(x)$

$$g(x)\cdot \Big| \quad \frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{with } \deg(r(x)) < \deg(g(x))$$

gives $f(x) = q(x)\cdot g(x) + r(x)$

<u>Ex</u> $f(x) = 3x^2 + 2x + 1$ and $g(x) = x - 2$

$$(\boxed{3x^2} + 2x + 1) : (\boxed{x} - 2) = \overset{3x^2 : x}{\overset{\parallel}{3x}} + \overset{8x : x}{\overset{\parallel}{8}} + \frac{17}{x-2}$$

$$
\begin{array}{l}
\underline{-(3x^2 - 6x \quad\;)} \quad \xleftarrow{\cdot (x-2)} \\
\qquad \boxed{8x} + 1 \\
\qquad \underline{-(8x - 16)} \quad \xleftarrow{\cdot (x-2)} \\
\qquad\qquad \boxed{17}
\end{array}
$$

is called the <u>remainder</u>

so $q(x) = 3x + 8$ and $r(x) = 17$

$$\underline{\text{check:}} \quad \left(3x+8+\frac{17}{x-2}\right)\cdot(x-2)$$

$$= (3x+8)\cdot(x-2) + \frac{17}{\cancel{x-2}}\cdot\cancel{(x-2)}$$

$$= 3x^2 - 6x + 8x - 16 + 17$$

$$= 3x^2 + 2x + 1 = f(x) \quad \text{- so ok!}$$

Two applications of polynomial division

A) To find asymptotes of rational functions

<u>Ex</u> $$\frac{3x^2+2x+1}{x-2} = 3x+8+\frac{17}{x-2}$$

has a vertical asymptote: the line $x=2$

and a non-vertical asymptote: the line $y=3x+8$

B) To factorise a polynomial as a product of degree 1 (linear) polynomials

<u>Ex</u> Factorise $x^3-4x^2-11x+30$ into linear factors.

<u>Solution</u> Three steps.

<u>Step I</u> Guess an integer root (zero)

[Note: has to divide 30]

I try $\underline{x=-3}$ and get

$(-3)^3 - 4\cdot(-3)^2 - 11\cdot(-3) + 30$

$= -27 - 36 + 33 + 30 = 0$

Then $(x-(-3)) = (x+3)$ is a factor

Step II Use polynomial division to find a polynomial of lower degree:

$(x^3 - 4x^2 - 11x + 30) : (x+3) \overset{\text{by poly. div.!}}{=} x^2 - 7x + 10$

Note: Remainder is 0.

This means $x^3 - 4x^2 - 11x + 30 = (x^2 - 7x + 10)\cdot(x+3)$

Step III We find the roots of $x^2 - 7x + 10$

They are $x = 2$, $x = 5$

so $x^2 - 7x + 10 = (x-2)\cdot(x-5)$

and $x^3 - 4x^2 - 11x + 30 = \underline{\underline{(x-2)\cdot(x-5)\cdot(x+3)}}$

Note 1 Not always possible to factorise

Ex $x^2 + 5$ has no roots!

$x^2 + 2x + 3$ —— " —— since $b^2 - 4ac$

$= 2^2 - 4\cdot1\cdot3 = 4 - 12 < 0$

Note 2 It can be difficult to guess roots!

– they don't have to be integers.

Start: 11.00

2. Rational and radical equations

A rational equation: $\dfrac{p(x)}{q(x)} = 0$

where $p(x)$ and $q(x)$ are polynomials.

Ex Equation $\dfrac{x+1}{(x-1)(x+3)} = 0$ then $x+1 = 0$

and $(x-1)(x+3) \neq 0$

so $\underline{\underline{x = -1}}$ i.e. $x \neq 1$, $x \neq -3$

Ex Probl. 10a from last week

$$1 + x + x^2 + \dots + x^{99} = 0 \qquad (*)$$

$$\frac{x^{100} - 1}{x - 1} = 0 \qquad (x \neq 1)$$

$$\text{so} \quad x^{100} = 1 \qquad (x \neq 1)$$

$$\text{so} \quad x = \pm \sqrt[100]{1} = \pm 1 \qquad (x \neq 1)$$

$$\underline{\underline{x = -1}}$$

check: $x = 1$ is not a solution of $(*)$

Ex. Eq. $\dfrac{x+1}{(x-1)(x+3)} = 2$

$$\frac{x+1}{(x-1)(x+3)} - 2 = 0$$

multiply -2 with $\dfrac{(x-1)(x+3)}{(x-1)(x+3)}$ which is 1

$$\frac{x+1 \quad - 2(x-1)(x+3)}{(x-1)(x+3)} = 0$$

$$\frac{(x+1) - 2(x^2+3x-x-3)}{(x-1)(x+3)} = 0$$

$$\frac{-2x^2 - 3x + 7}{(x-1)(x+3)} = 0$$

that is $\quad -2x^2 - 3x + 7 = 0$

with $\quad x \neq 1, \quad x \neq -3$

which you can solve.

Radical equations

- the unknown is under a root!

Ex $\quad 2\sqrt{x+1} = x - 2$

square both sides

$$4(x+1) = (x-2)^2 = (x-2)(x-2) = x^2 - 4x + 4$$

$$4x + \cancel{4} = x^2 - 4x + \cancel{4}$$

$$x^2 - 8x = 0$$

$$x(x-8) = 0$$

so $\underline{x = 0}$ or $\underline{x = 8}$

Note Not all of these need to be solutions of the original eq.

We have to test the candidates:

$\underline{x=0}$ l.h.s. $2\cdot\sqrt{0+1} = 2\cdot\sqrt{1} = 2$ not equal so $x=0$ is <u>not</u> a solution

r.h.s. $0-2 = -2$

$\underline{x=8}$ l.h.s. $2\cdot\sqrt{8+1} = 2\cdot\sqrt{9} = 2\cdot 3 = 6$ equal! so $\underline{\underline{x=8}}$ is the only solution.

r.h.s. $8-2 = 6$

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3. Inequalities

$-2 < -1$ read: 'minus two is <u>less than</u> minus one'

$\frac{1}{9} > \frac{1}{12}$ read: 'one ninth is <u>bigger than</u> one twelfth'

Also $\leq$, $\geq$

An inequality is claim that one expression (number) is less than, bigger than... another expression (number).

The solutions of an inequality are those values of x which make the claim true.

<u>Ex</u> $x-1 \geq 2$ is a claim

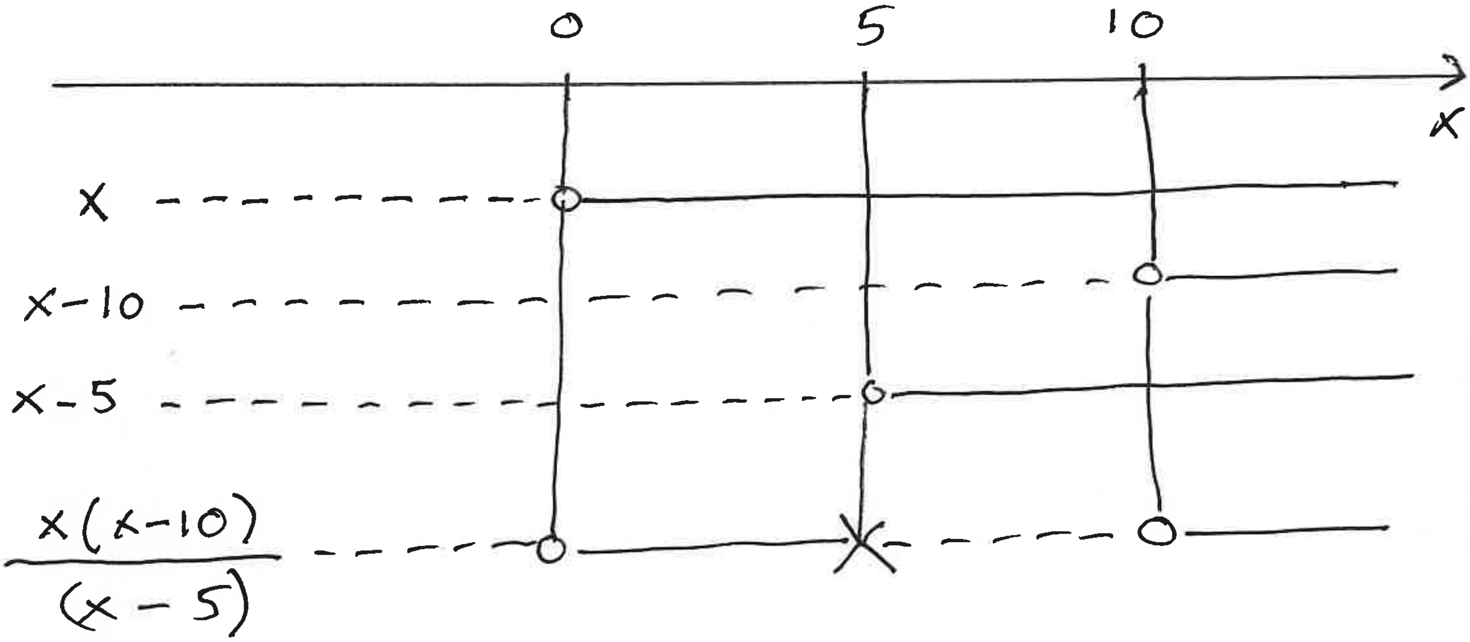
— is true if $x=5$ since $5-1 \geq 2$

— is not true if $x=2$ since $2-1 \geq 2$ is not true!

The solutions of the inequality are all the values of x such that $\underline{x \geq 3}$ - an infinite set of numbers.

<u>EX</u> Solve the inequality $\frac{x(x-10)}{x-5} \geq 0$

<u>Solution</u> Because we have 0 on the r.h.s. and factorised l.h.s. we can use a sign diagram.



that is $\underline{\underline{0 \leq x < 5 \text{ or } x \geq 10}}$

we also write $\underline{\underline{x \in [0, 5\rangle \text{ or } x \in [10, \infty\rangle}}$