EBA2911, lecture 2, 1 Sept. 2021, Runar Ile

Plan

1. Relative change and rate of change
2. Powers
3. Interest
4. Present value of cash flow

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1. Relative change and rate of change

$$\text{Relative change} = \frac{\text{new value} - \text{old value}}{\text{old value}}$$

Recall $\% = \frac{1}{100} = 0.01$

$3\% = 3 \cdot \frac{1}{100} = \frac{3}{100} = 0.03$

Ex Kåre's hourly wage increased from 163 kr to 181 kr. The relative change was

$$\frac{181\text{ kr} - 163\text{ kr}}{163\text{ kr}} = \frac{18}{163} = 11.0\,\%$$

$$\boxed{\text{Rate of change} = 1 + \text{relative change}}$$

Ex The rate of change in Kåre's hourly wage is $1 + 0.11 = 1.11$

Problem Last year Kåre earned 54000 with 163 kr/hour. If he works as much this year as last year how much would he earn (with the new wage)?

Solution $54000 \cdot 1.11 = \underline{\underline{59940}}$

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2. Powers

$$1.11^{3} = 1.11 \cdot 1.11 \cdot 1.11$$

$$1.11^{-3} = \frac{1}{1.11^{3}}$$

$$1.11^{\frac{2}{3}} = \sqrt[3]{1.11^{2}}$$

For integers m, n with $n > 0$ and a a number $a \geq 0$, then

$$a^{\frac{m}{n}} \overset{\text{definition}}{=} \sqrt[n]{a^{m}}$$

Calculate $1.11^{\sqrt{2}}$ on your calculator!

(answer: 1.159035....)

Answer 1.11 $\boxed{y^x}$ 2 $\boxed{\sqrt{x}}$ $\boxed{=}$

Same base:

$$2^{1.5} \cdot 2^{3.8} = 2^{1.5+3.8} = 2^{5.3}$$

Same exponent:

$$2^{4} \cdot 3^{4} = 2\cdot2\cdot2\cdot2 \cdot 3\cdot3\cdot3\cdot3$$
$$= 2\cdot3 \cdot 2\cdot3 \cdot 2\cdot3 \cdot 2\cdot3$$
$$= (2\cdot3)^4 = 6^{4}$$

Ex $\sqrt{2}\cdot\sqrt{3} = 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = 6^{\frac{1}{2}} = \sqrt{6}$

Pattern $a^r \cdot b^r = (ab)^r$

Problem Calculate 1.12^{-1} on the calc.

Solution 1: 1.12 $\boxed{y^x}$ 1 $\boxed{+/-}$ $\boxed{=}$

Solution 2: 1.12 $\boxed{1/x}$ (reason $1.12^{-1} = \frac{1}{1.12}$)

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3. Interest

Ex You deposit 40 000 into an account earning 2.3% annual interest
Interest is added after each year
(annual compounding of interest)
After a year the balance (what's on the account)
is $40\,000 + 40\,000 \cdot 2.3\%$

$$= 40\,000 \cdot (\underbrace{1 + 0.023}_{\text{growth factor}}) = \underline{\underline{40\,920.00}}$$

Problem What is the balance after 5 years ?

Solution

now

0 1 2 ...... 5 time line

$40000 \xrightarrow{\cdot 1.023} 40' \cdot 1.023 \xrightarrow{\cdot 1.023} 40' \cdot 1.023^2 \xrightarrow{\cdot 1.023^3} 40' \cdot 1.023^5$ (years

$$\underline{\underline{40000 \cdot 1.023^5}} = \underline{\underline{44\,816.52}}$$

$\boxed{\text{Start: } 11.03}$

Ex You deposit 40 000 with 2.3% nominal annual interest, but with quarterly compounding of interest.

The growth factor for one period (= 3 mnths) is

$$1 + \frac{2.3\%}{4} = 1 + \underbrace{0.575\%}_{\text{interest rate for one period}} = 1.00575$$

After 1 year the balance is

$$40000 \cdot 1.00575^4 = 40\,927.96$$

The annual growth factor is

$$1.00575^4 = 1.023199$$

The effective annual interest is

$$1.00575^4 - 1 = 0.023199 = 2,3199\%$$

nominal interest

Pattern $\quad B = B_0 \cdot \left(1 + \frac{r}{n}\right)^{m}$ — number of periods

balance after m periods — B

deposit (principal) — B_0

number of interest periods per year — n

Effective interest $\quad r_{eff} = \left(1+\frac{r}{n}\right)^n - 1$

4. Future and present value of a cash flow

Let K_0 be some investment / deposit / payment today. The future value K_n of K_0 in n years (or more generally n periods) with interest r is

$$K_n = K_0 \cdot (1+r)^n$$

The opposite: Suppose K_n will be paid n years (periods) from now with period interest r. Then the present value K_0 of K_n is given as

$$K_0 = \frac{K_n}{(1+r)^n}$$

Problem: 30 mill. is paid 5 years from now with 8% (annual) interest. Determine the present value.

Solution: $K_0 = \frac{30 \text{ mill}}{1.08^5} = \underline{\underline{20.42 \text{ mill.}}}$

"How much do you have to deposit today to have 30 mill 5 years from now with 8% annual interest"

Cash flow

Ex: You pay 20 mill today, and get paid back

6 mill after 3 years

7 mill after 4 years

8 mill after 5 years

with 8% interest

year	0	3	4	5
payment	-20	6	7	8

The present value of the cash flow is the sum of the present values of the payments:

$$-20 + \frac{6}{1.08^3} + \frac{7}{1.08^4} + \frac{8}{1.08^5} = \underline{\underline{-4.65}}$$

(bad investment!)

(„internrenten")

The internal rate of return is the interest which makes the present value of the cash flow $= 0$.

– hard to find in general.

Home work: Find the IRR of the cash flow above!

(answer: 1.1197%)