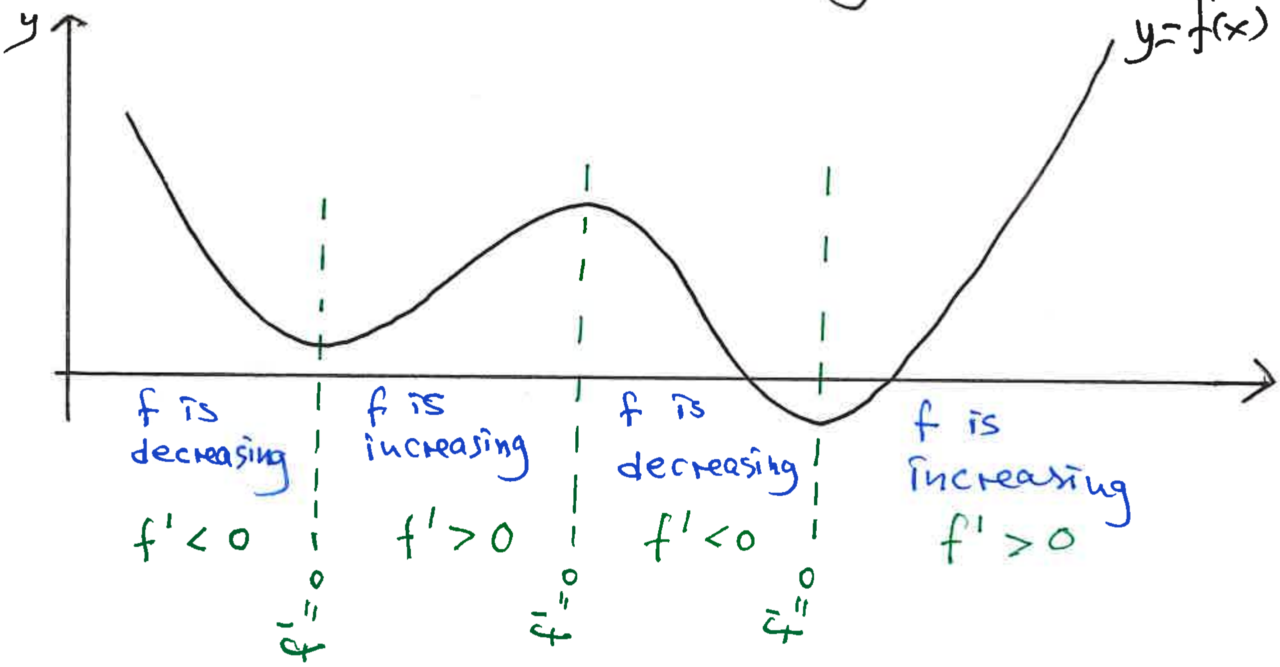
EBA2911, lecture 13, 3 Nov. 2021, Runar Ile

Plan
1. Local max/min and stationary points
2. Global max/min
3. The mean value theorem

1. Local max/min and stationary points



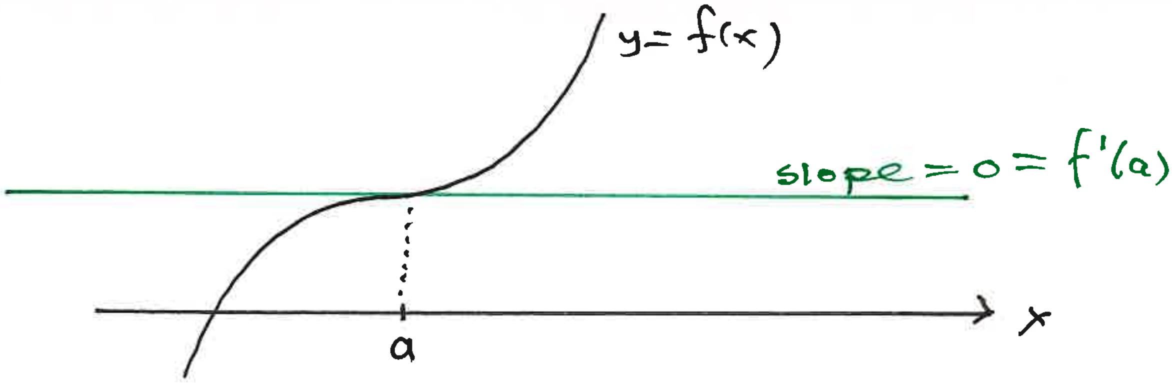
When $f'(x)$ is positive, the graph of $f(x)$ is increasing
When $f'(x)$ is negative, —— " —— decreasing

<u>Important conclusion</u> The sign diagram of $f'(x)$ determines where $f(x)$ is increasing and decreasing.

If $x = a$ is a local minimum point, then
$f'(a) = 0$ <u>and</u> $f'(x)$ changes sign from $-$ to $+$

If $x = a$ is a local maximum point, then
$f'(a) = 0$ <u>and</u> $f'(x)$ changes sign from $+$ to $-$

Ex



Here $x = a$ is neither a local max. point
nor a local min. point.
It is a terrace point

Definition If $f'(a) = 0$ then $x = a$ is a stationary point

Ex $f(x) = x^3 - 6x^2 + 5$. We find the stationary points.

- solve the equation $f'(x) = 0$

First we find $f'(x) = (x^3)' - 6(x^2)' + (5)'$

$$= 3x^2 - 6 \cdot 2x + 0$$

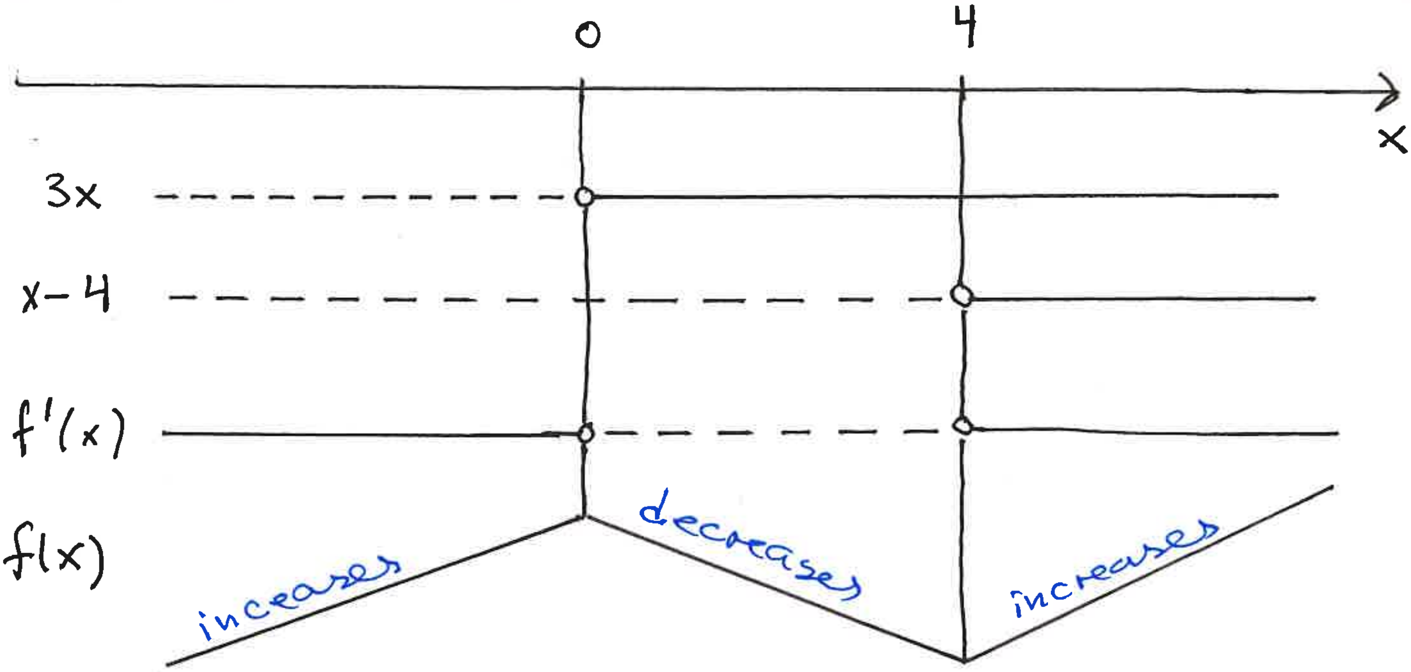
$$= 3x^2 - 12x$$

$$= 3x(x - 4)$$

so $f'(x) = 0$ has solutions $x = 0$, $x = 4$

Where is $f(x)$ increasing / decreasing?

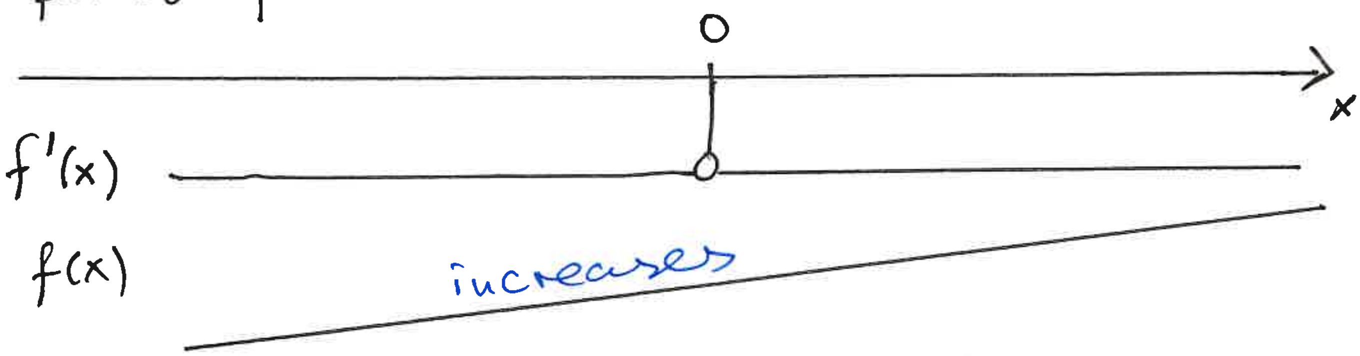
We determine the sign of $f'(x)$ by a sign diagram.



$f(x)$ is strictly increasing for $x \leq 0$ (so $x \in (\leftarrow, 0]$)
$f(x)$ is strictly decreasing for $0 \leq x \leq 4$ (so $x \in [0, 4]$)
$f(x)$ is strictly increasing for $x \geq 4$ (so $x \in [4, \rightarrow)$)

Then $x = 0$ is a local maximum point
and $x = 4$ is a local minimum point

Ex $f(x) = x^3 + 1$
$f'(x) = 3x^2$, so $x = 0$ is a stationary point for $f(x)$.



Conclusion: $f(x)$ is strictly increasing for all x on the number line

Start: 10.55

2. Global max/min

> **The extreme value theorem** If $f(x)$ is a continuous function on the interval $D_f = [a, b]$ then $f(x)$ has a global maximum and a global minimum

Possible max/min points:

(*) stationary points $(f'(x) = 0)$

(*) cusp points (where $f'(x)$ is not defined)

(*) end points (a and b)

Ex $f(x) = x^3 - 6x^2 + 5$ and $D_f = [-1, 7]$

Find max/min of $f(x)$.

(*) stationary points: $f'(x) = 3x^2 - 12x = 0$

gives $\underline{x = 0}$, $\underline{x = 4}$

(*) cusp points: non ($f'(x)$ is defined everywhere)

(*) end points: $\underline{x = -1}$, $\underline{x = 7}$.

These four points are my candidate points for max/min.

Calculate:

$f(-1) = -2$ $\quad$ $f(4) = -27$

$f(0) = 5$ $\quad$ $f(7) = 54$

So $x = 4$ gives the global minimum $f(4) = \underline{\underline{-27}}$ and $x = 7$ gives the global maximum $f(7) = \underline{\underline{54}}$

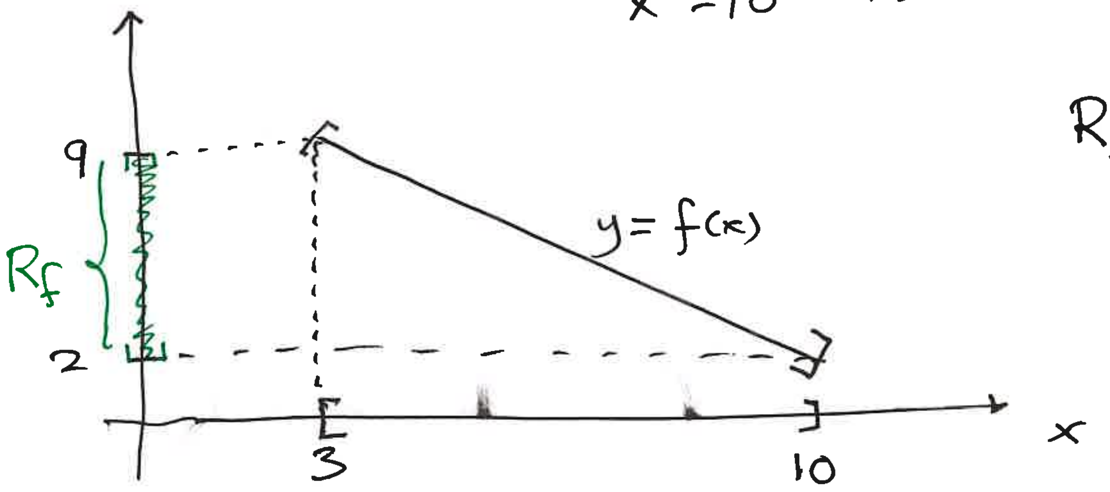
Ex $f(x) = 12 - x$ and $D_f = [3, 10]$

(*) $f'(x) = -1 \neq 0$ so no stationary points

(*) no cusps

(*) end points: $x = 3$ is a max. point

$x = 10$ is a min. point

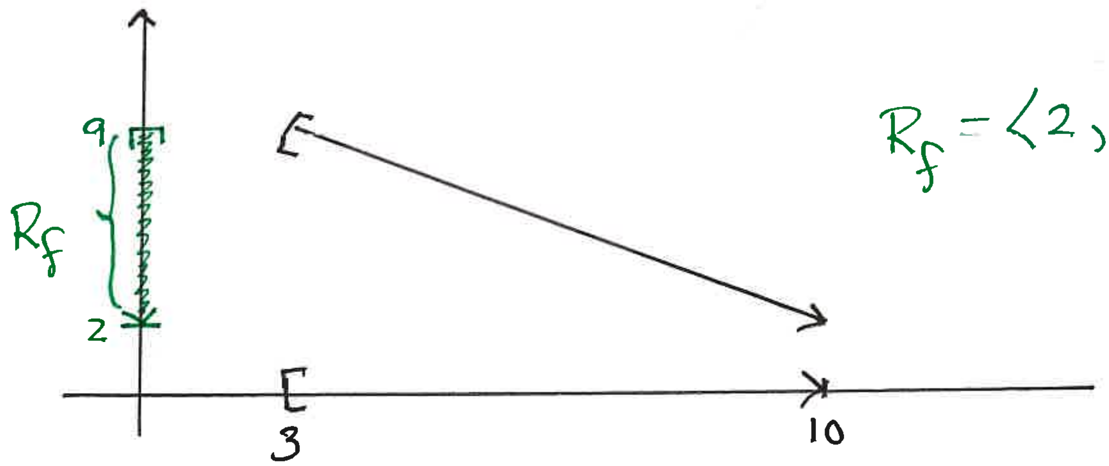


$R_f = [2, 9]$

Ex $f(x) = 12 - x$ and $D_f = [3, 10\rangle$

$x = 3$ is still the maximum point

but there is no minimum point.

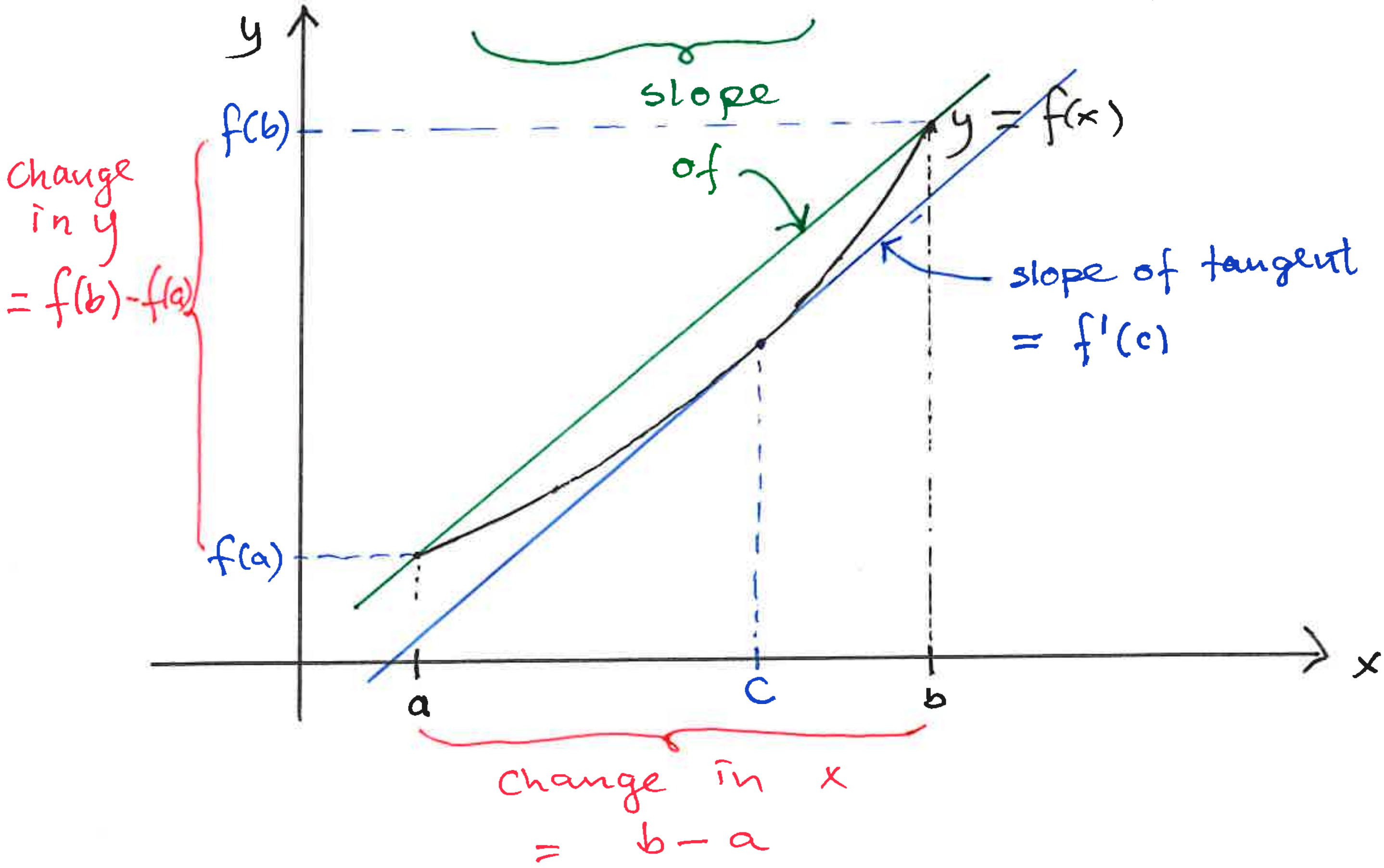


$R_f = \langle 2, 9]$

3. The mean value theorem

If $f(x)$ is continuous in the interval $[a, b]$ and differentiable (no cusps) then there is a number c between a and b $(a < c < b)$ such that

$$f'(c) = \underbrace{\frac{f(b) - f(a)}{b - a}}_{\text{slope of}} = \frac{\text{change in } y}{\text{change in } x}$$



<u>Ex</u> $f(x) = e^x + x^2$. Then $f(0) = 1$ and $f(1) = e + 1$

By the mean value thm. there is a number c between 0 and 1 such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{e+1-1}{1} = e$$

<u>Note</u> $f'(x) = e^x + 2x$ (easy) but we cannot solve the eq. $e^x + 2x = e$ (no exact solution)