EBA 2911, lecture 1, 25 Aug. 2021, Runar Ile

Plan

1. Intro. to the Course
2. Algebraic expressions
3. Roots
4. Powers
5. Order of operations
6. Absolute value

1. Intro. to the Course

Autumn

- Financial math.
- Functions and graphs
- Differentiation and optimization

Spring

- Integration
- Systems of linear equations
- Functions in two variables

$$z = f(x, y)$$

2. Algebraic expressions

Variables: $x, y, z, x_1, x_2, x_3, \ldots$

$a, b, c, \ldots, m, n$

Multiply with a number

$$3 \cdot x \overset{\text{short writing}}{=} 3x$$

$$3 \cdot 2 \neq 32$$

$$\sqrt{3} \cdot x = \sqrt{3}x$$

$$(-1) \cdot x = -x$$

$$1 \cdot x = x$$

$$0 \cdot x = 0$$

Addition $\quad x + x = 2x$

$x + y \quad$ no simplification

$x + y + x = 2x + y$

Multiplication $\quad x \cdot y = xy$

$x \cdot x = x^2$

$xy \cdot x^2 = x \cdot y \cdot x \cdot x = x^3 y$

Dividing
$$\underbrace{\frac{x + 4y}{z} \, , \quad \frac{2xy + \sqrt{5}}{3x + y^2}}$$

Rational expressions: fractions of polynomials

Other expressions: $\sqrt{x^2+1}$, $\dfrac{3\sqrt{x}+1}{\sqrt{x}-1}$

We can insert numbers for the variables:

Ex $\quad \dfrac{2y}{x^2+1}$ with $x = 3$, $y = -1$

gives a number: $\dfrac{2 \cdot (-1)}{3^2+1} = \dfrac{-2}{10} = -\dfrac{1}{5} = -0.20$

If $x = 1$, $y = 3$, then $\dfrac{2 \cdot 3}{1^2+1} = \dfrac{6}{2} = 3$

But $\dfrac{2y}{x^2+1}$ cannot be simplified further.

Problem We have the rational expression

$$\frac{x^2 - x - 6}{x - 3}$$

a) Fill in

x	1	5	-2	2	8	3
$\frac{x^2-x-6}{x-3}$	3	7	0	4	10	"$\frac{0}{0}$" undefined

b) Find the pattern.

Add two to the x-value (except $x=3$)

Shorter: $x + 2 \quad (x \neq 3)$ Start 11.00

Quadratic expansion

$$(x+r)^2 = x^2 + 2rx + r^2$$

Ex: $(x+5)^2 = x^2 + 10x + 25$

Ex: $13^2 = (10+3)^2 = 10^2 + 2\cdot 3 \cdot 10 + 3^2$
$= 100 + 60 + 9 = \underline{169}$

Conjungate expansion

$$(x-r)(x+r) = x^2 - r^2$$

Ex $(x-5)(x+5) = x^2 - 25$

Ex $8 \cdot 12 = (10-2)(10+2) = 10^2 - 2^2 = 96$

3. Roots

The square root of 5 is the positive number a such that $a \cdot a = 5$.

(a is in the calculator $a = 2.2361\ldots$)

We write a as $\sqrt{5}$

Note: Negative numbers don't have square roots.

$\sqrt{0} = 0$

Problem Compute (without calc.)

a) $(\sqrt{2}+3)^2 = (\sqrt{2})^2 + 2\sqrt{2}\cdot 3 + 3^2 = \underline{\underline{11 + 6\sqrt{2}}}$

b) $(\sqrt{5}-1)(\sqrt{5}+1) = (\sqrt{5})^2 - 1^2 = 5 - 1 = \underline{\underline{4}}$

There are other roots:

$\sqrt[3]{5}$ is the number a such that $a \cdot a \cdot a = 5$

$\sqrt[5]{32} = 2$ (since $2\cdot 2\cdot 2\cdot 2\cdot 2 = 32$)

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4. Powers – repeated multiplication

Ex: $3\cdot 3\cdot 3\cdot 3 = 3^4$ " three to the power of four "

$4\cdot 4\cdot 4 = 4^3$

exponent

$4^3 \neq 4\cdot 3$

$\shortparallel$ 64 $\qquad$ $\shortparallel$ 12

base

$$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^5$$
$$= 10^{2+3}$$

so $a^n \cdot a^m = a^{n+m}$

$$\frac{3^6}{3^4} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{3 \cdot 3}{1} = 3^2$$
$$= 3^{6-4} \qquad \left(\text{so } 3^{-4} = \frac{1}{3^4}\right)$$

$$1 = \frac{5^3}{5^3} = 5^{3-3} = 5^0$$

$$(a^n)^m = a^{n \cdot m}$$

Ex $(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$
$$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$
$$= 3^8$$
$$= 3^{2 \cdot 4}$$

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5. Order of operations

Problem Compute

a) $2 + 3 \cdot 4 \overset{?}{=} \begin{cases} 5 \cdot 4 = 20 \\ 2 + 12 = 14 \end{cases} = (2+3) \cdot 4$

b) $2 \cdot 2^2 = \begin{cases} 2 \cdot 4 = 8 \\ 4^2 = 16 \end{cases} = (2 \cdot 2)^2$

Problem $-5^2 \overset{?}{=} \begin{cases} 25 & = (-5)^2 \\ -25 & \end{cases}$ $\quad -x^2$

Why? $-5^2 = (-1)\cdot 5^2 = -25$

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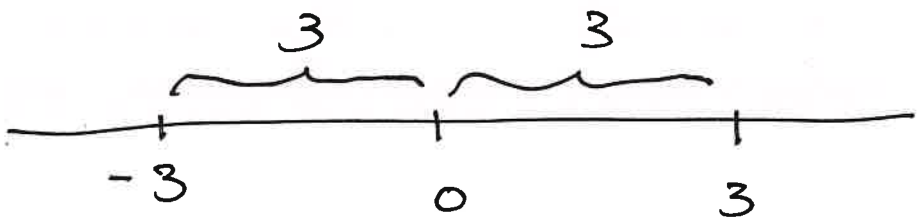
6. Absolute value

If a is a number, then $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

"the absolute value of a"

Ex $|3| = 3$, $|-3| = -(-3) = 3$

$|a|$ = distance between 0 and a on the number line



Problem Simplify $\sqrt{x^2}$

Solution If $x \geq 0$ then $\sqrt{x^2} = x$

If $x < 0$ then $\sqrt{x^2} = -x$

In short: $\sqrt{x^2} = |x|$.

Ex $\sqrt{(x-5)^2} = |x-5|$, $\sqrt{(-3)^2} = \sqrt{9} = 3$