EBA2911 Mathematics for Business Analytics autumn 2020

Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

R. Lucas

Lecture 8

on Wednesday 30 Sept. 10-11.45 in B2-060

Sec. 6.3.1-3, 5.4-5, 4.7

Increasing/decreasing functions. Circles, ellipses. Polynomial functions.

Here are recommended exercises from the textbook [SHSC].

Section **6.3** exercise 3
Section **5.4** exercise 1, 3
Section **5.5** exercise 1-6
Section **4.7** exercise 4

Problems for the exercise session
Wednesday 30 Sept. at 12-15 in CU1-067 or on Zoom

Problem 1 Determine the equations of the circles in figure 1.

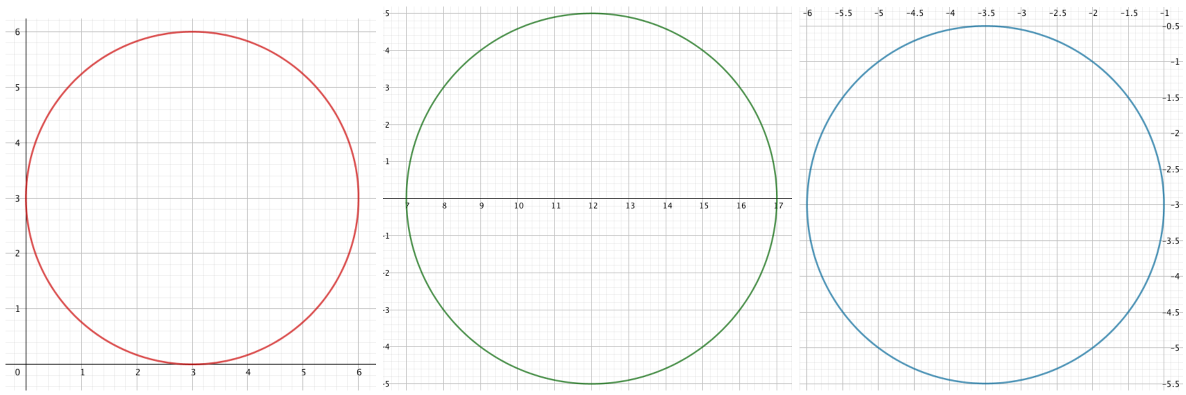

Figure 1: Circles a-c

Problem 2 Determine the center S and the radius r of the circles.

a) $(x-3)^2+(y-4)^2=5$ b) $(x+1)^2+y^2=3$ c) $(3x-2)^2+(3y-4)^2=9$

d) $x^2+y^2-4x-10y=-25$ e) $x^2+y^2+6x-12y=-44$

f) $25x^2+25y^2-20x-30y=-12$

Problem 3 Determine the equations of the ellipses in figure 2.

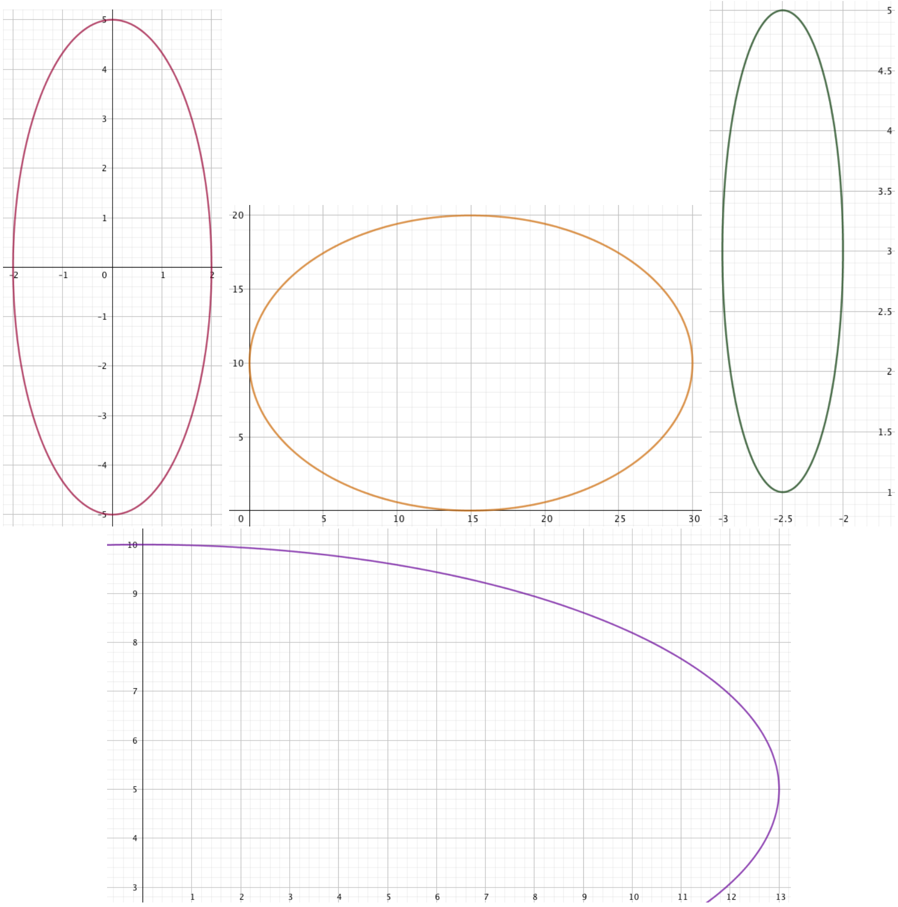

Figure 2: Ellipses a-d

Problem 4 Determine the center S and the semi-axes of the ellipse. Draw a sketch of the ellipse.

a) $\dfrac{x^2}{9}+\dfrac{y^2}{16}=1$ b) $\dfrac{(x-1)^2}{9}+\dfrac{(y-2)^2}{16}=1$ c) $16(x-1)^2+9(y-2)^2=144$

d) $\dfrac{x^2}{2}+y^2-6y=-8$ e) $9x^2+18x+4y^2=27$ f) $4x^2+9y^2-16x+18y=11$

g) $25x^2+4y^2-100x-40y=-100$

Problem 5 Give elementary arguments for the statements.
a) $f(x)=x^2$ with $x\geqslant 0$ is strictly increasing.
b) $f(x)=\sqrt{x}$ is strictly increasing.
c) $f(x)=\frac{1}{x}$ with $x>0$ is strictly decreasing.

Problem 6 Determine the intersection points of
a) the line $3x+2y=12$ and the line $-3x+2y=-6$
b) the line $2x+y=6$ and the ellipse in Problem 4a

Problem 7 Determine which expressions (below) and graphs (in figure 3) which belong together.
1) $x^4-8x^3+24x^2-32x+\dfrac{161}{10}$
2) $\dfrac{x^5}{10}-\dfrac{3x^4}{2}+\dfrac{17x^3}{2}-\dfrac{45x^2}{2}+\dfrac{137x}{5}-10$
3) $-x^3+6x^2-11x+7$
4) $x^4-10x^3+35x^2-50x+26$

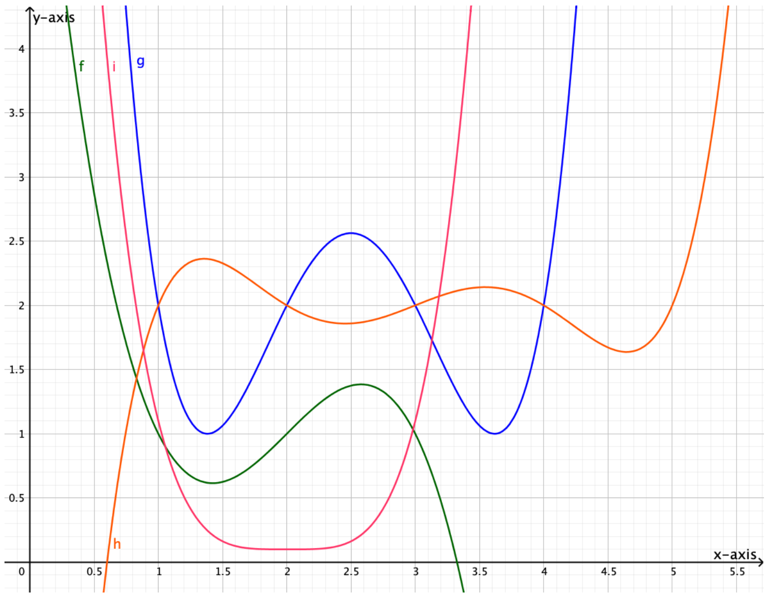

Figure 3: The graphs of four polynomial functions

Answers

Problem 1

a) $(x-3)^2+(y-3)^2=9$ b) $(x-12)^2+y^2=25$ c) $(x+3{,}5)^2+(y+3)^2=6{,}25$

Problem 2

a) $S=(3\,,4)$, $r=\sqrt{5}$ b) $S=(-1\,,0)$, $r=\sqrt{3}$ c) $S=(\frac{2}{3}\,,\frac{4}{3})$, $r=1$

d) $S=(2\,,5)$, $r=2$ e) $S=(-3\,,6)$, $r=1$ f) $S=(\frac{2}{5}\,,\frac{3}{5})$, $r=\frac{1}{5}$

Problem 3

a) $\frac{x^2}{4}+\frac{y^2}{25}=1$ b) $\frac{(x-15)^2}{225}+\frac{(y-10)^2}{100}=1$ c) $4(x+2{,}5)^2+\frac{(y-3)^2}{4}=1$

d) $\frac{x^2}{169}+\frac{(y-5)^2}{25}=1$

Problem 4

a) $S=(0\,,0)$, semi-axes $a=3$, $b=4$

b) $S=(1\,,2)$, semi-axes $a=3$, $b=4$

c) $S=(1\,,2)$, semi-axes $a=3$, $b=4$

d) $S=(0\,,3)$, semi-axes $a=\sqrt{2}$, $b=1$

e) $S=(-1\,,0)$, semi-axes $a=2$, $b=3$

f) $S=(2\,,-1)$, semi-axes $a=3$, $b=2$

g) $S=(2\,,5)$, semi-axes $a=2$, $b=5$

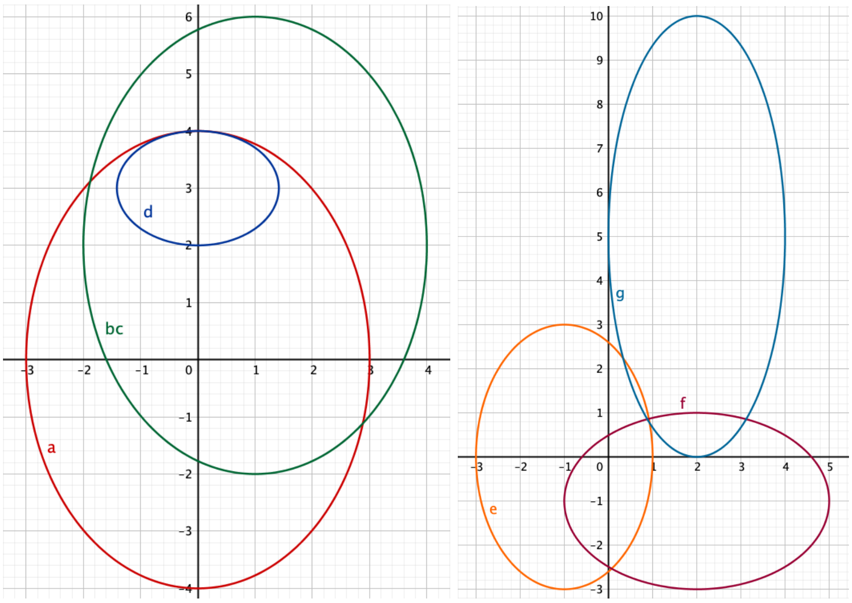

Figure 4: Ellipses a-d and e-g

Problem 5

a) Suppose $0 \leqslant x_1 < x_2$. Then $x_2 = x_1 + k$ for a positive constant k. Then $f(x_2) = (x_1 + k)^2 = x_1^2 + 2kx_1 + k^2$. The product and the sum of two positive numbers are positive numbers, hence $2kx_1 + k^2$ is a positive number. Then $f(x_1) = x_1^2 < x_1^2 + 2kx_1 + k^2 = f(x_2)$ and $f(x) = x^2$ for $x \geqslant 0$ is strictly increasing.

b) We divide each side of the inequality $x_1 < x_2$ with the positive number x_2 and get the inequality $\frac{x_1}{x_2} < 1$. The square root of a number which is less than 1 is itself less than 1, i.e. $\sqrt{\frac{x_1}{x_2}} < 1$. But $\sqrt{\frac{x_1}{x_2}} = \frac{\sqrt{x_1}}{\sqrt{x_2}}$. We get the inequality $\frac{\sqrt{x_1}}{\sqrt{x_2}} < 1$ and when we multiply each side with the positive number $\sqrt{x_2}$ we get the inequality $f(x_1) = \sqrt{x_1} < \sqrt{x_2} = f(x_2)$. Hence $f(x) = \sqrt{x}$ strictly increasing.

c) We divide each side of the inequality $x_1 < x_2$ with the positive number x_2 and get the equivalent inequality $\frac{x_1}{x_2} < 1$. Then we divide this inequality by the positive number x_1 and get $f(x_2) = \frac{1}{x_2} < \frac{1}{x_1} = f(x_1)$. Hence $f(x) = \dfrac{1}{x}$ for $x > 0$ is strictly decreasing.

Problem 6

a) $(3\,,\,\frac{3}{2})$

b) $(3\,,0)$ and $(\frac{15}{13}\,,\,\frac{48}{13})$

Problem 7

- $f(x) = -x^3 + 6x^2 - 11x + 7$
- $g(x) = x^4 - 10x^3 + 35x^2 - 50x + 26$
- $h(x) = \dfrac{x^5}{10} - \dfrac{3x^4}{2} + \dfrac{17x^3}{2} - \dfrac{45x^2}{2} + \dfrac{137x}{5} - 10$
- $i(x) = x^4 - 8x^3 + 24x^2 - 32x + \dfrac{161}{10}$