EBA2911, lecture 8, 30 Sept. 2020, Runar Ile

Plan
1. Increasing and decreasing functions
2. Circles and ellipses
3. Polynomial functions

1. Increasing and decreasing functions

Ex $f(x) = 0.03x^2 + 8x - 1500$, $D_f = [0, \rightarrow\rangle$
(meaning: $x \geq 0$)

Is $f(x)$ increasing?
Is $f(x)$ decreasing?
- or neither?

Can look at the graph (use GeoGebra or similar)

or: complete the square and draw the graph by hand.

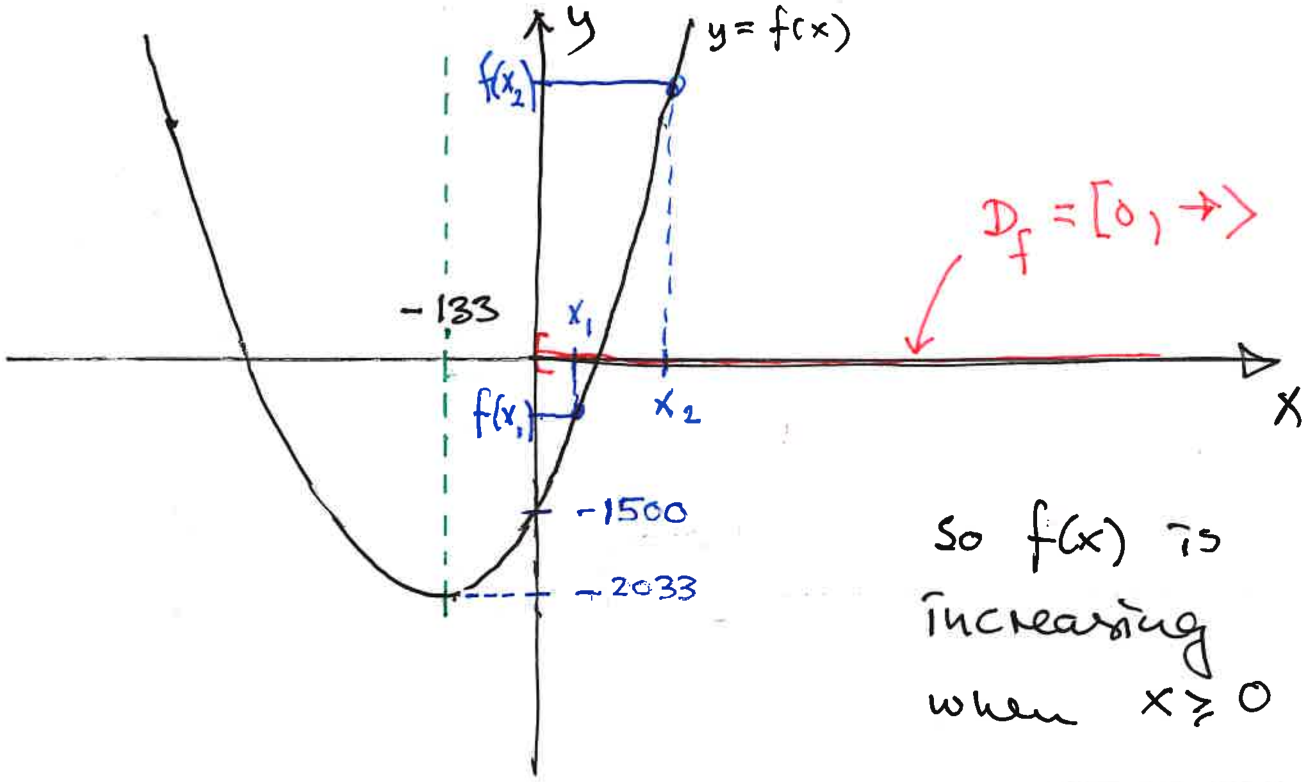
$$f(x) = 0.03\left[x^2 + \frac{800}{3}x\right] - 1500$$

$$= 0.03\left[\left(x + \frac{800}{6}\right)^2 - \left(\frac{800}{3}\right)^2\right] - 1500$$

$$= 0.03\left(x + \frac{800}{6}\right)^2 - \frac{6100}{3}$$

Symmetry axis: $x = -\frac{800}{6} \approx -133$ (y free)

Minimum value: $y = f\left(-\frac{800}{6}\right) = -\frac{6100}{3} \approx -2033$



So $f(x)$ is increasing when $x \geq 0$

<u>Definition</u> A function $f(x)$ is <u>increasing</u> if for all $x_1 < x_2$ one has

$$f(x_1) \leq f(x_2)$$

<u>Ex</u> $f(x) = 2x + 5$ is increasing for all x!

Reason: If $x_1 < x_2$

multiply by 2 on each side

$$2x_1 < 2x_2$$

add 5 to each side

$$f(x_1) = 2x_1 + 5 < 2x_2 + 5 = f(x_2)$$

and so $f(x)$ is (strictly) increasing

Definition A function $f(x)$ is **decreasing** if for all $x_1 < x_2$ one has

$$f(x_1) \geq f(x_2)$$

Problem Show that $f(x) = -2x + 5$ is (strictly) decreasing.

Solution Suppose $x_1 < x_2 \quad \Big| \cdot (-2)$

$$-2x_1 > -2x_2$$

Add 5 to each side:

$$f(x_1) = -2x_1 + 5 > -2x_2 + 5 = f(x_2)$$

and so $f(x)$ is (strictly) decreasing.

Problem We have the constant function $f(x) = 5$. Decide whether $f(x)$ is increasing/decreasing/neither.

Solution

Increasing: If $x_1 < x_2$ then $f(x_1) = 5 \leq 5 = f(x_2)$

Decreasing: If $x_1 < x_2$ then $f(x_1) = 5 \geq 5 = f(x_2)$

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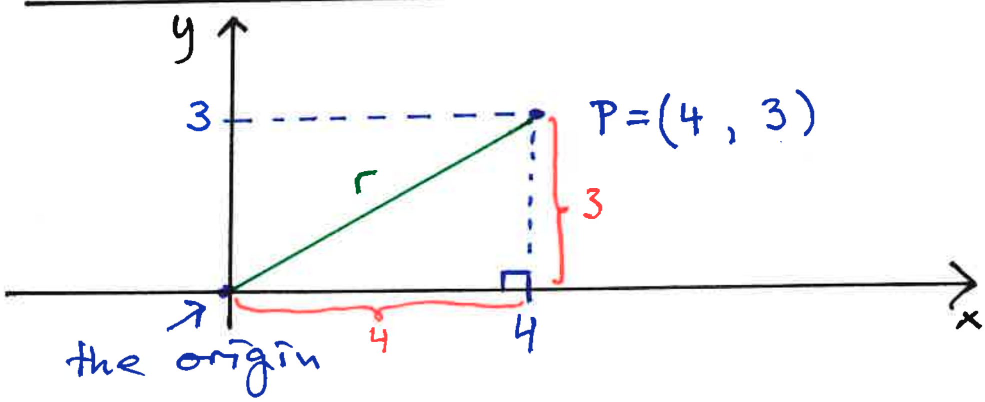
2. Circles and ellipses

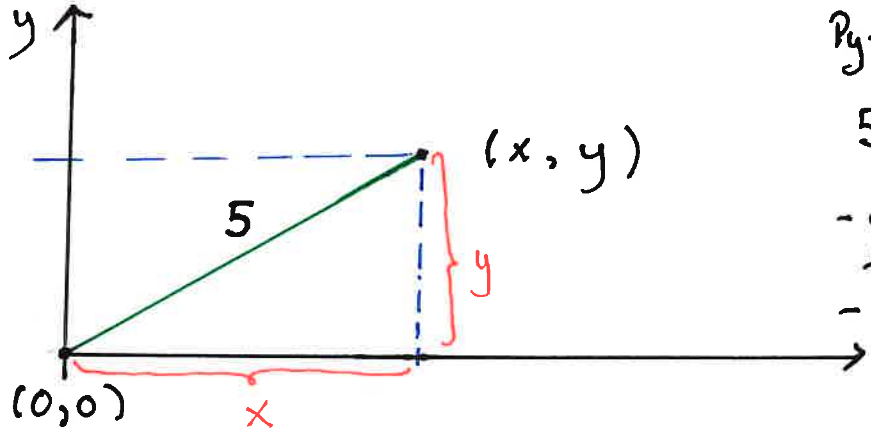


What is the distance from P to the origin? Pythagoras gives the answer:

$$r^2 = 4^2 + 3^2 \quad (r \geq 0)$$

$$r^2 = 16 + 9 = 25$$

$$r = \sqrt{25} = 5$$



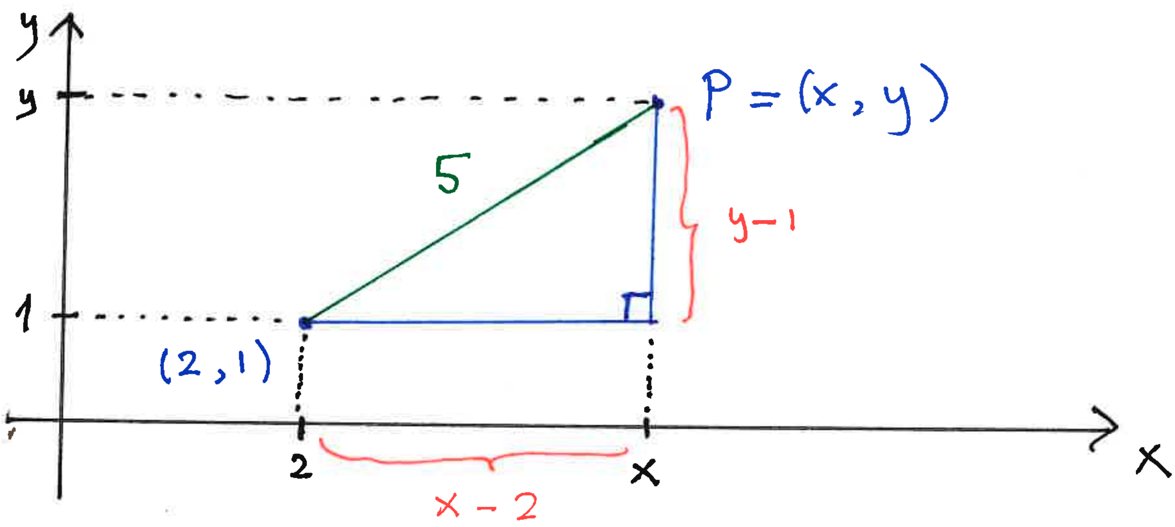
Pythagoras:

$$5^2 = x^2 + y^2$$

- one equation, two unknowns
- infinitly many solutions

The solutions are all points on the circle with origin as centre and with radius 5.

<u>Ex</u> What is the equation of the points on the circle with $(2,1)$ as centre and radius 5?



Pythagoras: $5^2 = (x-2)^2 + (y-1)^2$

$$25 = x^2 - 4x + 4 + y^2 - 2y + 1$$

that is: $x^2 + y^2 - 4x - 2y = 20$

<u>Problem</u> Determine the radius and the centre of the circle.

a) $x^2 + (y+5)^2 = 10$ b) $x^2 + y^2 - 2x + 6y = -9$

Solutions

a) Centre: $(0,-5)$, radius: $\sqrt{10}$

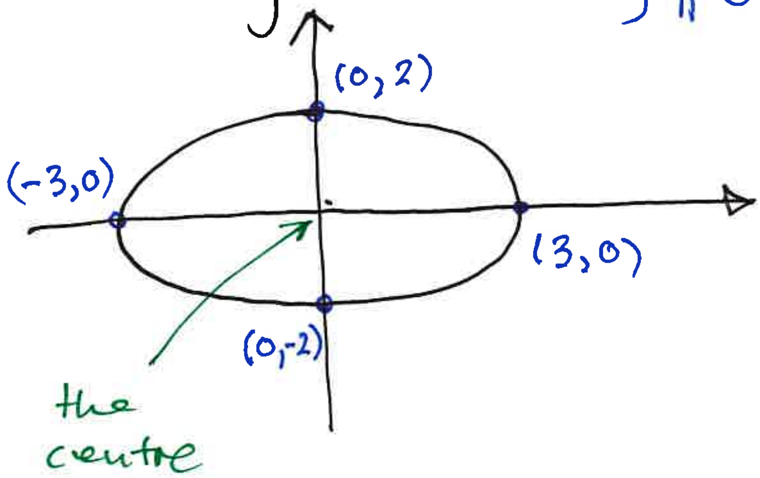
b) $\underbrace{(x-1)^2}_{x^2-2x+1} + \underbrace{(y+3)^2}_{y^2+6y+9} = -9+1^2+3^2 = 1$

Centre: $(1,-3)$, radius: $\sqrt{1} = 1$

Ellipses

Ex $\quad 4x^2+9y^2=36$

x	3	-3	0	0
y	0	0	2	-2



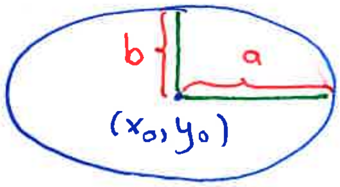
Divide the eq. by 36:

$$\overset{\frac{1}{9}}{\frac{4}{36}} \cdot x^2 + \overset{\frac{1}{4}}{\frac{9}{36}} \cdot y^2 = 1$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

Reminds us of the circle equation but the x-axis is stretched by a factor 3 and the y-axis ———— " ———— 2

In general, any ellipse is the set of solution of an equation which can be written as



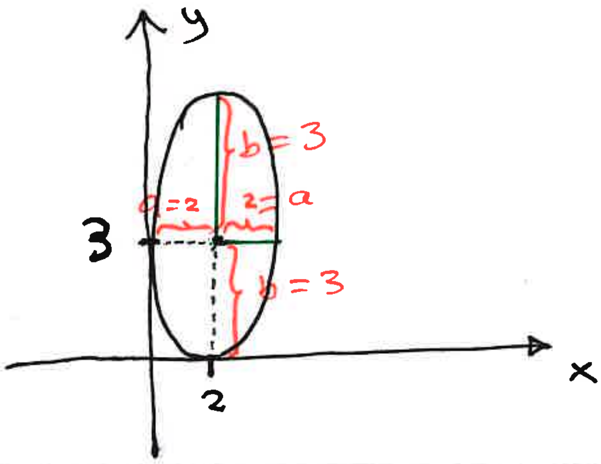
$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

Here (x_0, y_0) is the centre of the ellipse and a and b are the semi-axes

<u>Ex</u> $$\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$$

centre: $(2, 3)$

semi-axes: $a = \sqrt{4} = 2$, $b = \sqrt{9} = 3$



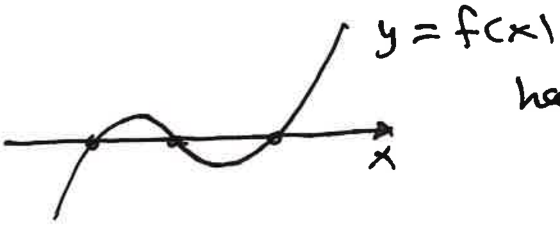
3. Polynomial functions

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

is a polynomial function of degree n if $a_n \neq 0$

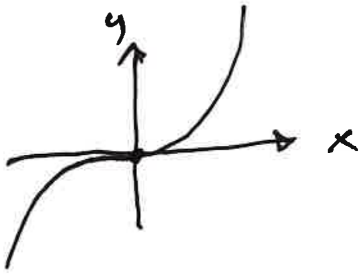
It has a maximum of n roots (zeros)

Ex

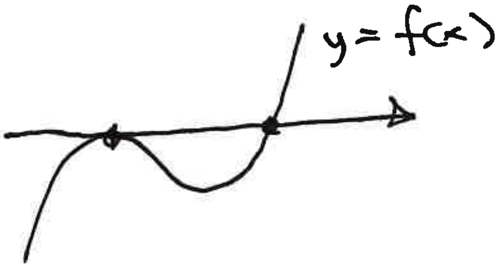


has degree 3

Ex $f(x) = x^3$



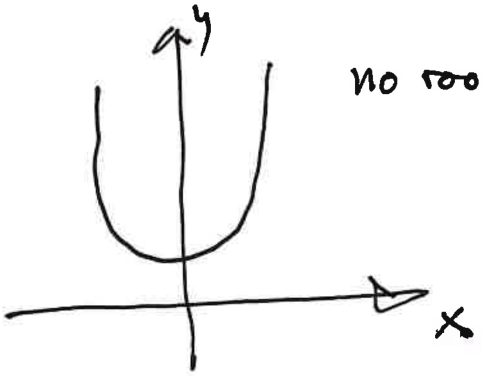
Ex



has degree 3

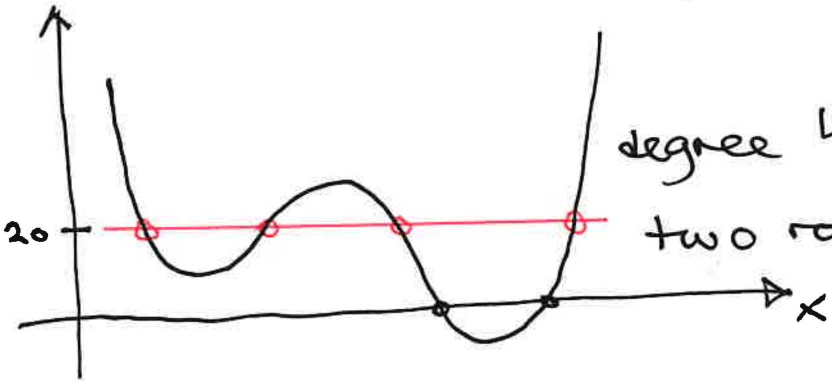
Odd degree : at least one root

Ex $f(x) = x^4 + 1$



no roots

Ex:



degree 4

two roots

$f(x) = 20$ has 4 roots

equivalently: $\underbrace{f(x) - 20}_{\text{still a polynomial, of the same degree as } f(x).} = 0$ has the same 4 roots.