EBA2911, lecture 5, 9 Sept 2020, Rumar Ile

1. Linear and quadratic equations
2. Equations with parametres: the abc-formula
3. Completing the square
4. Equations with given solutions

1. Linear and quadratic equations

A linear expression $ax + b$ (a and b are numbers, $a \neq 0$)

Ex $4x - 3$ $(a = 4, b = -3)$

A linear equation: An equation which can be transformed into an equivalent equation of the form $ax + b = 0$ $(a \neq 0)$

Ex The equation $\frac{1}{x+3} = \frac{2}{x+4}$

• $(x+3)(x+4)$

Multiply with a common denominator on each side.

can be transformed:

$$x+4 = 2(x+3)$$

distr. law: $x + 4 = 2x + 6$

subtract $2x - 6$: $-x - 2 = 0$ $(a = -1, b = -2)$

$(x \neq -3, x \neq -4)$

A quadratic expression: $ax^2 + bx + c$

where a, b, c are numbers and $a \neq 0$

An quad. eq.: - an eq. which can be transformed to an eq. $ax^2 + bx + c = 0$

EX The equation $3x+9 = (x-1)(x+3)$

- resolve the parantheses so that we can collect terms

$$3x+9 = x^2+3x-x-3$$

subtract $3x+9$ on each side

$$x^2-x-12 = 0 \qquad (a=1,\ b=-1,\ c=-12)$$

EX The equation $\frac{1}{x}+\frac{2}{x+1} = 3 \quad \Big| \cdot x(x+1)$

$$x+1+2x = 3x(x+1)$$

$$3x+1 = 3x^2+3x$$

subtr. $3x+1$ on each side

$$3x^2-1 = 0 \qquad (a=3,\ b=0,\ c=-1)$$

$$(x \neq 0,\ x \neq -1)$$

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2. Equations with parametres: the abc-formula

If $a \neq 0$ the quadratic formula (abc-formula) gives the solutions to any quadratic eq. on standard form: $ax^2+bx+c=0$

Then $$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Ex $3x^2 + 4x - 5 = 0$ $(a = 3, b = 4, c = -5)$

The quad. formula gives

$$x = \frac{-4 \pm \sqrt{4^2 - 4\cdot 3\cdot(-5)}}{2\cdot 3}$$

$$= \frac{-4 \pm \sqrt{16 + 60}}{6} = \frac{-4 \pm \sqrt{76}}{6}$$

$$= \frac{-4 \pm \sqrt{4.\ 19}}{6} = \frac{-\cancel{4}^{2} \pm \cancel{2}\sqrt{19}}{\cancel{6}\ 3}$$

$$= -\frac{2}{3} \pm \frac{\sqrt{19}}{3}$$

Three cases: $b^2 - 4ac > 0$ gives two solutions

$b^2 - 4ac = 0$ gives one solution

$b^2 - 4ac < 0$ no solutions

Problem Determine the number of solutions of the equations.

a) $x^2 + 5x + 4.6 = 0$ $\quad 5^2 - 4\cdot 1\cdot 4.6 > 0$: two sol's

b) $-x^2 + 2x - 1 = 0$ $\quad 2^2 - 4\cdot(-1)\cdot(-1) = 0$: one sol.

c) $4x^2 - 5x - 5 = 0$ $\quad (-5)^2 - 4\cdot 4\cdot(-5) > 0$: two sol's.

The quadratic formula is often inefficient:

Ex Equation: $-3x^2 + 7 = 0$ $(a=-3,\ b=0,\ c=7)$

$$-3x^2 = -7 \quad | : (-3)$$

$$x^2 = \frac{-7}{-3} = \frac{7}{3}$$

$$|x| = \sqrt{x^2} = \sqrt{\frac{7}{3}}$$

so $x = \pm\sqrt{\frac{7}{3}}$

Ex Equation $2x^2 - 6x = 0$ $(a=2,\ b=-6,\ c=0)$

$$2(x^2-3x) = 0 \quad | : 2$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

Either $x = 0$ or $x - 3 = 0$

$x = 3$

Pattern

If $a \cdot b = 0$ then $a = 0$ or $b = 0$

(or both)

3. Completing the square

Ex Equation $x^2 + 6x - 16 = 0$

Claim: $x^2 + 6x = (x+3)^2 - 9$

- because $(x+3)^2 = x^2 + 6x + 9$

so $(x+3)^2 - 9 = x^2 + 6x$

$$(x+3)^2 - 9 - 16 = 0$$

so $(x+3)^2 = 25$

so $x + 3 = 5$ or $x + 3 = -5$

$\underline{\underline{x = 2}}$ or $\underline{\underline{x = -8}}$

Problem Solve the quad. eq. by completing the square

a) $x^2 - 8x - 33 = 0$

b) $x^2 + 2x = 63$

Solutions

a) $\frac{-8}{2} = -4$ so $x^2 - 8x = (x-4)^2 - 4^2$

(because $(x-4)^2 = x^2 - 2 \cdot 4x + 4^2$)

Rewrite eq: $(x-4)^2 - 4^2 - 33 = 0$

so $(x-4)^2 = 49$

so $x - 4 = 7$ or $x - 4 = -7$

$\underline{\underline{x = 11}}$ or $\underline{\underline{x = -3}}$

b) $x^2 + 2x = (x+1)^2 - 1^2$

so rewrite eq: $(x+1)^2 - 1 = 63$

so $(x+1)^2 = 64$

so $x+1 = 8$ or $x+1 = -8$

$\underline{\underline{x = 7}}$ or $\underline{\underline{x = -9}}$

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4. Equations with given solutions

If r_1 and r_2 are solutions ('roots') to the quadratic equation $x^2+bx+c=0$

then $(x-r_1)(x-r_2) = x^2 - r_2x - r_1x + r_1r_2$

$= x^2 - (r_1+r_2)x + r_1r_2$

and $b = -(r_1+r_2)$ and $c = r_1r_2$.

Ex $x^2 + 6x - 16 = (x-2)(x+8)$

Problem Determine the expression x^2+bx+c with the given roots (zeros)

a) 1 and 2 : $(x-1)(x-2) = x^2 - 3x + 2$

b) 11 and -3 : $(x-11)(x+3) = x^2 - 8x - 33$

Ex: $3(x-1)(x-2) = 3x^2 - 9x + 6$