EBA2911, lecture 3, 1 Sept. 2020, Runar Ile

Plan: 1. Total present value of cash flow

2. Series

3. Annuities

1. Total present value of cash flow

Present value of an amount (K) paid n years (or periods) from now with interest r.

= what you have to deposit today (K_0) for the balance to become K in years from now if the interest is r.

Since $K = K_0 \cdot (1+r)^n$

so that $$K_0 = \frac{K}{(1+r)^n}$$

Ex 50000 (K) 3 years from now with 4% interest has present value

$$K_0 = \frac{50\,000}{1.04^3} = \underline{\underline{44\,449.82}}$$

That is: If you deposit 44449.82 today into an account earning 4% interest, the balance will be 50000 three years from now.

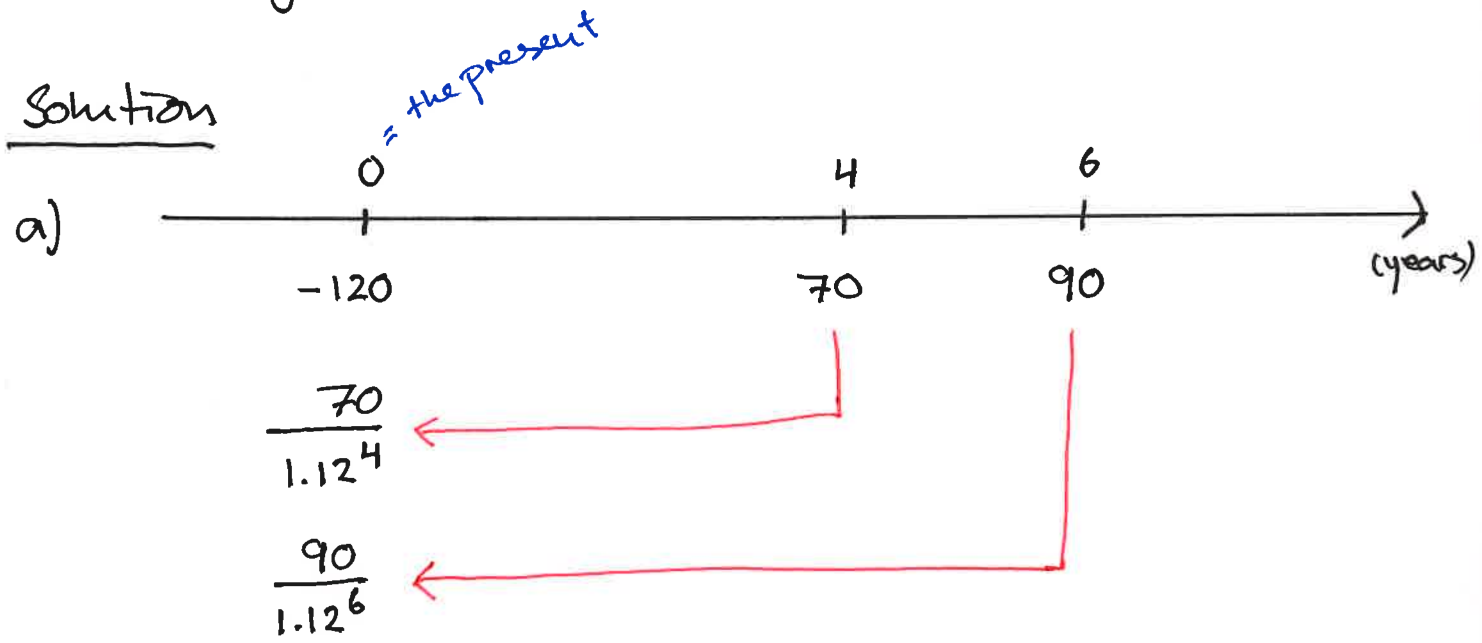
Ex An investment of 120 mill is supposed to give payments of 70 mill 4 years from now and 90 mill 6 years from now. Suppose the interest is 12 %.

a) Determine the total present value of the cash flow.

b) Do you think this is a good investment?

Solution

a)



= total present value of the cash flow

$$= -120 + \frac{70}{1.12^4} + \frac{90}{1.12^6} = \underline{\underline{-29.92}}$$

b) We don't get 12 % interest on this investment.

In fact (trying, plotting ...) the internal rate of return (= the intereset that makes the tot. pres. value = 0) is (approx.) 5.81 % because

$$-120 + \frac{70}{1.0581^4} + \frac{90}{1.0581^6} = 0.0$$

5,81% can be interpreted as annual yield on this investment.

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2. Series

– many terms added

Ex $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{100}$ is a series

a term

with 10 terms.

We write $a_1 + a_2 + a_3 + \ldots + a_{10}$

the third term

the 10^{th} term

Geometric series $a_1 + a_2 + \ldots + a_n$

where each term is k times the previous term.
(k is a number!)

$$a_2 = k \cdot a_1$$

$$a_3 = k \cdot a_2 = k \cdot k \cdot a_1 = k^2 \cdot a_1$$

$$a_4 = k \cdot a_3 = k \cdot k^2 \cdot a_1 = k^3 \cdot a_1$$

$$\vdots$$

$$a_{10} = k^9 \cdot a_1$$

We can find a short expression for this series:

$$a_1 + a_2 + \ldots + a_n = a_1 + ka_1 + k^2 a_1 + k^3 a_1 + \ldots + k^{n-1} a_1$$

$$= a_1 \underbrace{(1 + k + k^2 + k^3 + \ldots + k^{n-1})}_{\frac{k^n - 1}{k-1}}$$

$$= a_1 \cdot \frac{k^n - 1}{k - 1}$$

<u>Problem</u> Compute the sum

$$5 + 5\cdot 1.003 + 5\cdot 1.003^2 + 5\cdot 1.003^3 + \dots + 5\cdot 1.003^{60}$$

<u>Solution</u> This is a geometric series with

$a_1 = 5$, $k = 1.003$ and $n = 61$

So the sum is $5\cdot\dfrac{1.003^{61}-1}{1.003-1} = 5\cdot\dfrac{1.003^{61}-1}{0.003}$

$= \underline{\underline{334.14}}$

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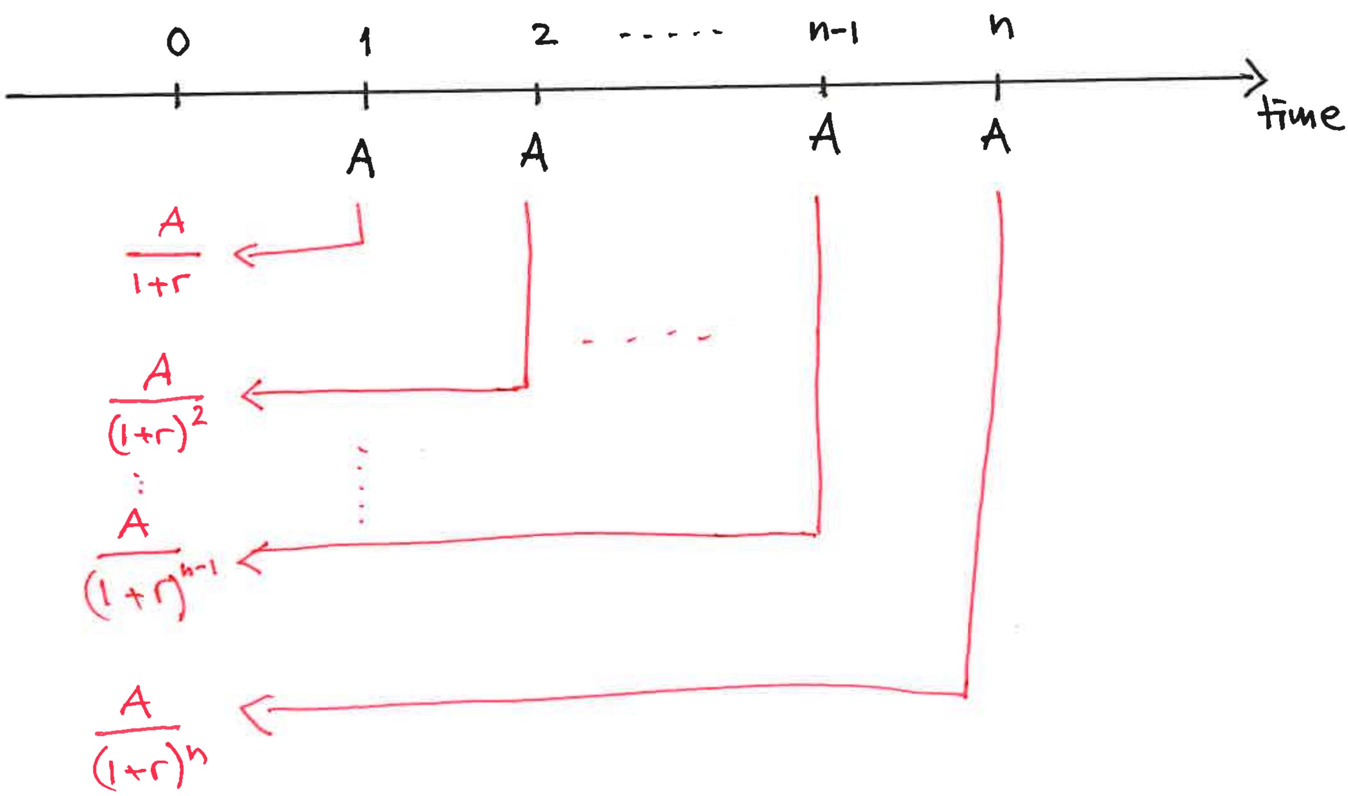
4. Annuities – regular cash flows



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the sum is the total present value of the regular cash flow. This is a geometric series with $a_1 = \dfrac{A}{1+r}$, the number of terms $= n$ and $k = \dfrac{1}{1+r}$. The sum is then (by the formula) $\dfrac{A}{1+r}\cdot\dfrac{\left(\frac{1}{1+r}\right)^n - 1}{\frac{1}{1+r} - 1}$

But a finite geometric series is also a geometric series in the opposite direction! Then

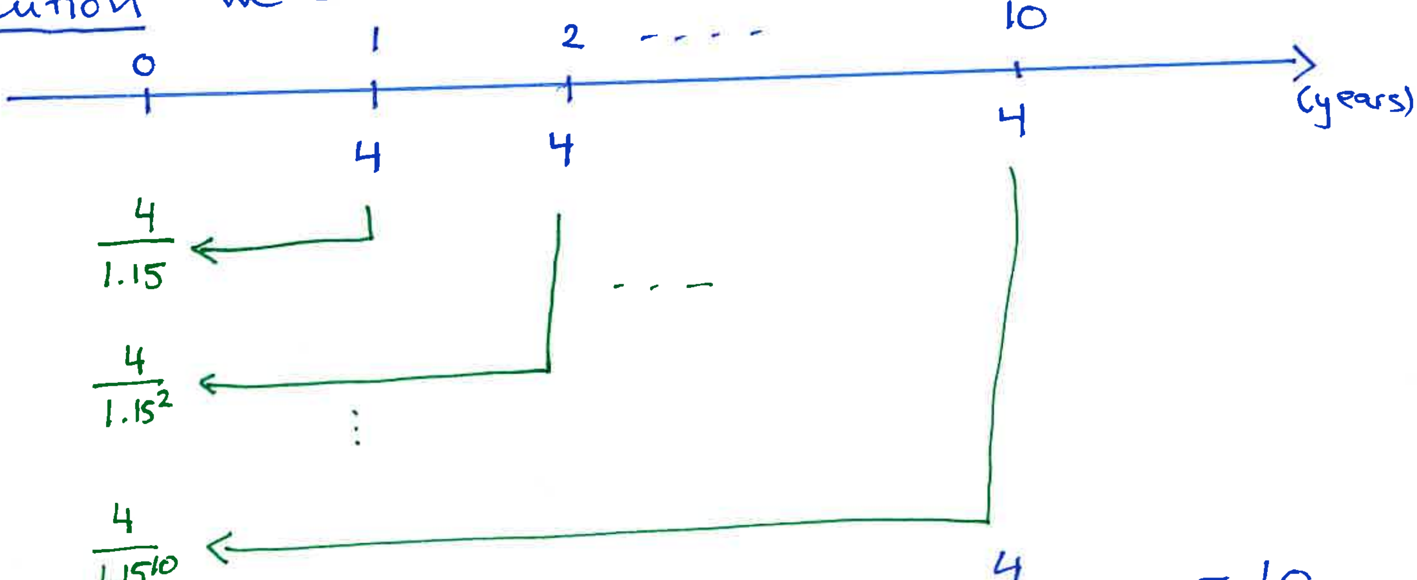
$$a_1 = \frac{A}{(1+r)^n}, \quad \text{the number of terms} = n, \quad k = 1+r.$$

Then the sum is

$$\frac{A}{(1+r)^n} \cdot \frac{(1+r)^n - 1}{r}$$

Problem Hege considers an investment where 4 mill is paid every year for 10 years. The first payment is one year from now. Suppose the discount rate is 15%. What is a fair price for this cash flow?

Solution We determine the tot. pres. val. of the cash flow.



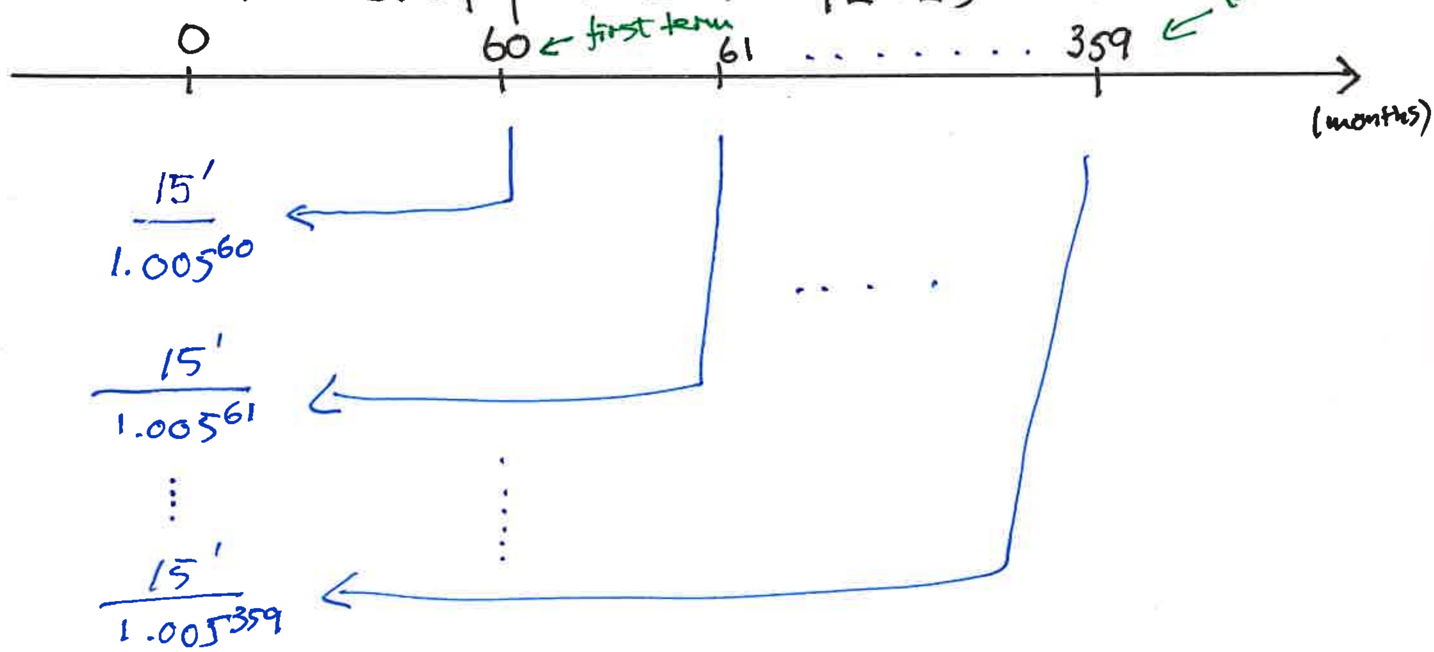
the sum is a geom. series with $a_1 = \frac{4}{1.15^{10}}$, $n = 10$ and $k = 1.15$, the formula gives $\frac{4}{1.15^{10}} \cdot \frac{1.15^{10}-1}{0.15} = \underline{\underline{20.08}}$

Ex (Term paper 2019 a, probl. 6a)

Kåre considers a mortgage with monthly payments running for 25 years. The first payment is 5 years from now. Kåre reckons he can pay 15 000 every month. The interest is 6%. Determine the geometric series which gives the tot. pres. val. of the cash flow. Calc. how much Kåre can borrow.

Solution The period rate is $\frac{6\%}{12} = 0,5\%$

Number of periods is $12 \cdot 25 = 300$



The sum is a geom. ser. with

$$a_1 = \frac{15'}{1.005^{359}}, \quad n = 300, \quad k = 1.005$$

The tot. pres. val.: $\frac{15000}{1.005^{359}} \cdot \frac{1.005^{300} - 1}{0.005} = \underline{\underline{1734620.76}}$

(what Kåre can borrow)