EBA 2911, lecture 10, 7 Oct. 2020, Runar Ile

Plan
1. Inverse functions
2. Exponential functions
3. Logarithms

1. Inverse functions

- expression
- a table of function values
- a graph
- a situation (emperical function)

Ex $f(x) = (x-3)^2$.
with domain $D_f = [3, \rightarrow\rangle$
(so $x \geq 3$)

Table of function values

x	3	4	5	6	7	$g(x)$
$f(x)$	0	1	4	9	16	x

← the inverse function of $f(x)$.

so $g(0) = 3$, $g(1) = 4$, $g(4) = 5$ ....

$f(g(0)) = f(3) = 0$ and $g(f(3)) = g(0) = 3$
$f(g(1)) = f(4) = 1$ and $g(f(4)) = g(1) = 4$
$f(g(4)) = f(5) = 4$ and $g(f(5)) = g(4) = 5$

Definition $f(x)$ with domain D_f and $g(x)$ with domain D_g are inverse functions if

$f(g(x)) = x$ for all x in D_g and $g(f(x)) = x$ for all x i D_f

Here the domain of $g(x)$ is the range of $f(x)$. So : $D_g = R_f$

Also: $f(x)$ is the inverse function of $g(x)$

so $R_g = D_f$

How to find an expression for the inverse function?

① Solve the equation $y = f(x)$ for x

② Switch the variables x and y.

③ Put $D_g = R_f$, which has to be determined.

Ex $f(x) = (x-3)^2$ with $D_f = [3, \rightarrow\rangle$

① We solve the equation

$$y = (x-3)^2 \quad \text{for } x.$$

– take the square root on each side

$$\sqrt{y} = |x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ -x+3 & \text{if } x \leq 3 \end{cases}$$

so $\sqrt{y} = x-3$ since $x \in D_f = [3, \rightarrow\rangle$

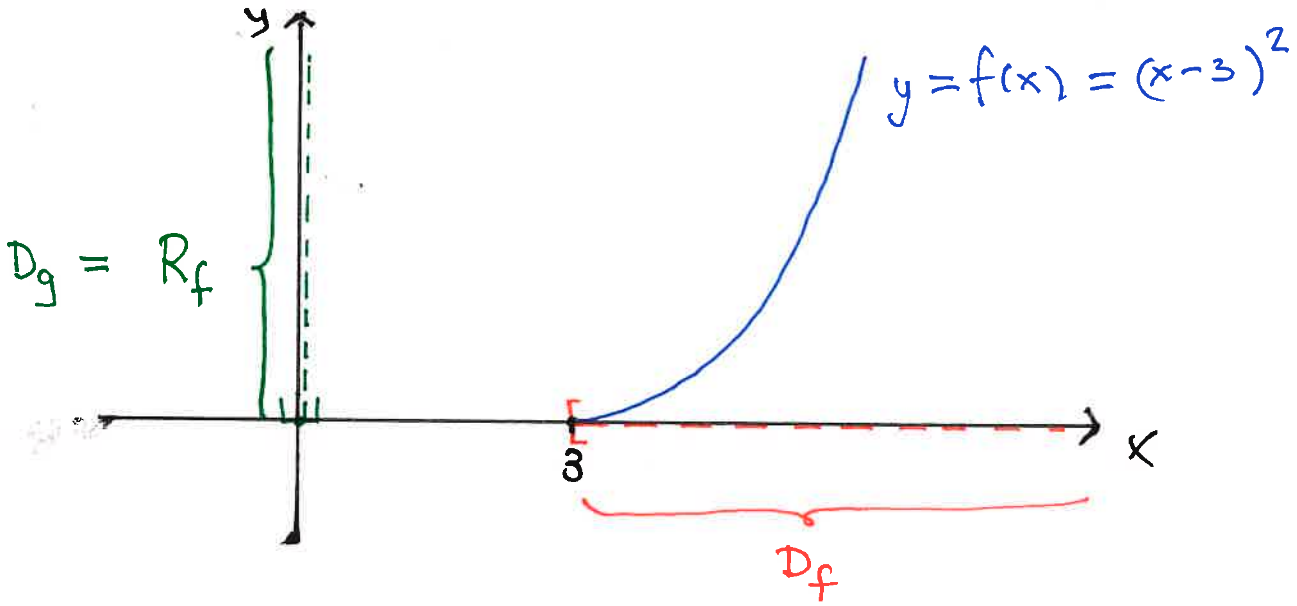
and then $x = \underline{3+\sqrt{y}}$

② Switch variables: $y = g(x) = 3+\sqrt{x}$

(3) $D_g = R_f = [0, \rightarrow\rangle$ because

$f(x) = (x-3)^2 = y$ has a solution

for $x \geq 3$ for all values $y \geq 0$



Note that $f(g(x)) = \left((3+\sqrt{x}) - 3\right)^2 = x$

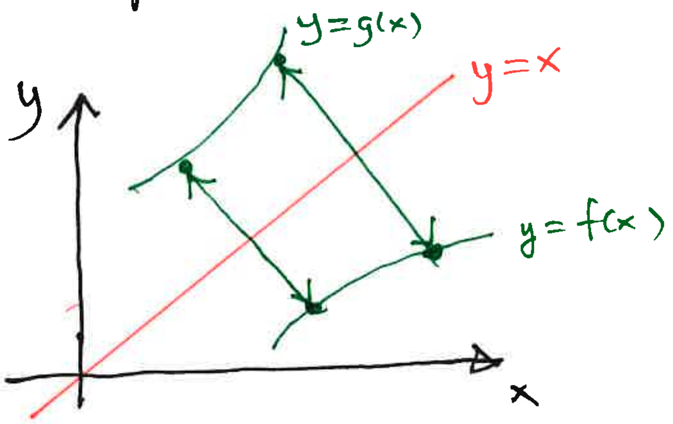
and $g(f(x)) = 3 + \sqrt{(x-3)^2} = x$ (since $x \geq 3$)

We start at 1100

The graph of the inverse function

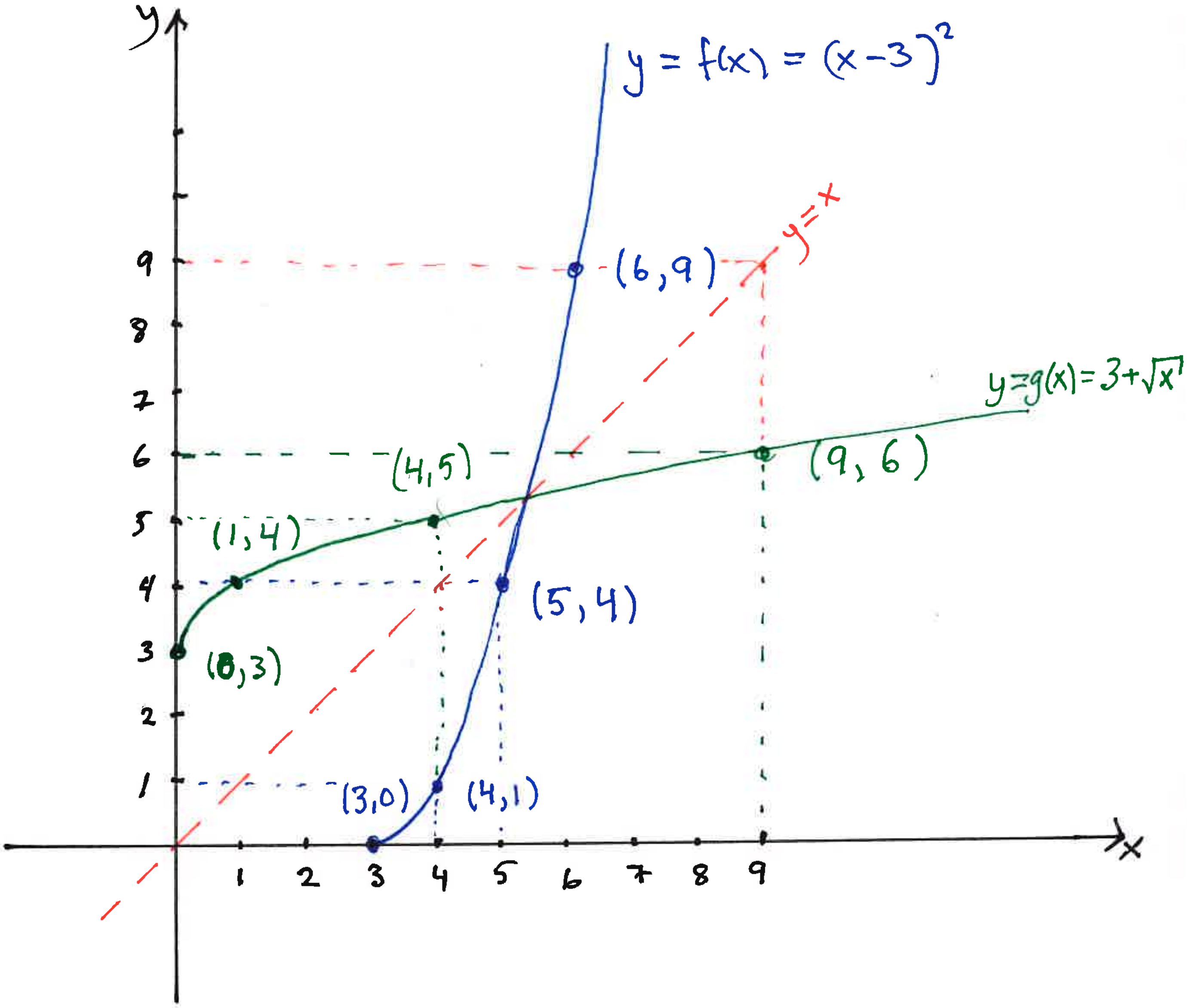
– is the mirror image of the graph of $f(x)$

with respect to the "diagonal" $y = x$

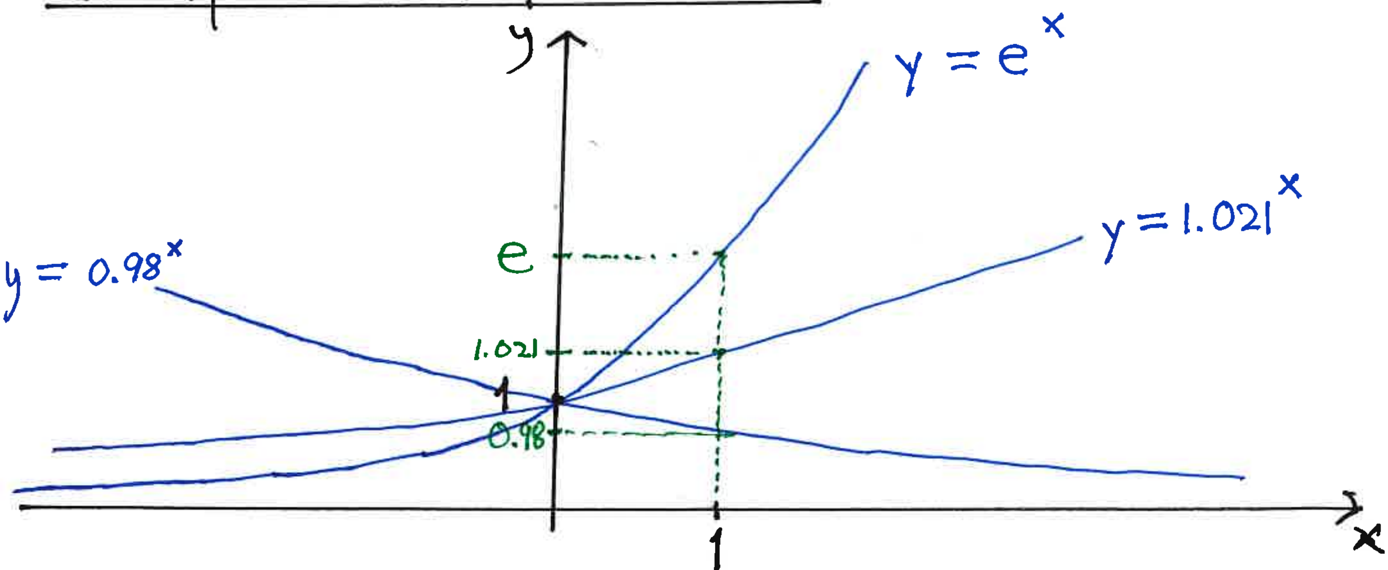


Ex $f(x) = (x-3)^2$ with $D_f = [3, \rightarrow\rangle$

x	3	4	5	6	7	$g(x)$
$f(x)$	0	1	4	9	16	x



2. Exponential functions



$\underline{a > 1}$ $f(x) = a^x$ is strictly increasing without bounds

and $a^x \xrightarrow[x \to -\infty]{} 0^+$

($a^{-1000} = \dfrac{1}{a^{1000}}$ is very close to 0)

$\underline{0 < a < 1}$ $f(x) = a^x$ is strictly decreasing

and $a^x \xrightarrow[x \to \infty]{} 0^+$

Note : a is always positive

Both cases D_f = all numbers

and $R_f = \langle 0, \rightarrow \rangle$

Powerrules If $f(x) = a^x$ then

$$f(x) \cdot f(y) = a^x \cdot a^y = a^{x+y} = f(x+y)$$

and $\dfrac{1}{f(x)} = \dfrac{1}{a^x} = a^{-x} = f(-x)$.

3. Logarithms

Suppose $a > 0$ and $a \neq 1$

Then $g(x) = \log_a(x)$ is the inverse function of $f(x) = a^x$ and

$D_g = R_f = \langle 0, \to \rangle$

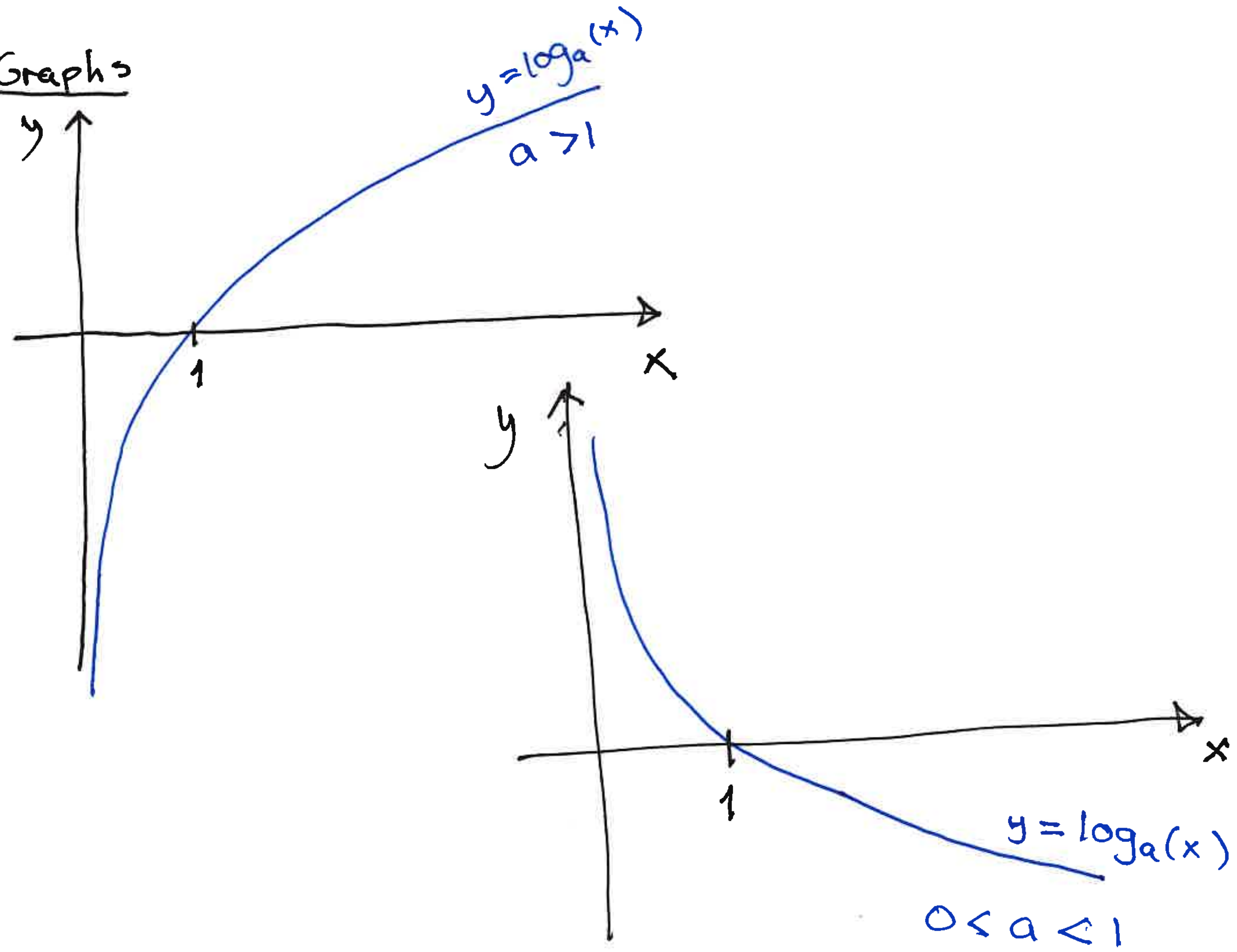
a is called the base of the logarithm.

<u>Ex</u> $a = 2$, $\log_2(10)$ = the number which 2 has to be raised to to give 10

$2^{3.322} \approx 10$

so $\log_2(10) \approx 3.322$

Graphs



Rules $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

$$\log_a(x^r) = r \cdot \log_a(x)$$

Definition $\ln(x) = \log_e(x)$ e = Euler number

– is called the natural logarithm.