EBA 2911, lecture 1, 19 Aug. 2020, Runar Ile

Plan:
1. Intro. to the Course
2. Algebraic expressions
3. Roots
4. Powers
5. Order of operations
6. Absolute value

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1. Intro. to the Course

Autumn

- Financial math.
- Functions and graphs
- Differentiation and optimization

Spring

- Integration
- Systems of linear equations
- Functions in two variables $z = f(x, y)$.

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2. Algebraic expressions

Variables: $x, y, z, x_1, x_2, x_3, \ldots$

$a, b, c, \ldots, m, n$

Multiply with a number

$$3 \cdot x \overset{\text{short writing}}{=} 3x = x + x + x$$

$$3 \cdot 2 \neq 32$$

$$\sqrt{3} \cdot x = \sqrt{3}x$$

$$(-1) \cdot x = -x$$

$$1 \cdot x = x$$

$$0 \cdot x = 0$$

Addition: $x + x = 2x$

$x + y$ no simplification

$x + y + x = 2x + y$

Multiplication: $x \cdot y = xy$

$x \cdot x = x^2$

$xy \cdot x^2 = x \cdot y \cdot x \cdot x = x^3 y$

Dividing $\dfrac{x + 4y}{z}$, $\dfrac{2xy + \sqrt{5}}{3x + y^2}$

Rational expressions: Fractions of polynomials

Other expressions: $\sqrt{x^2+1}$, $\dfrac{3\sqrt{x} + 1}{\sqrt{x} - 1}$

We can insert numbers for the variables:

Ex $\dfrac{2y}{x^2+1}$ with $x = 3$, $y = -1$

gives a number: $\dfrac{2 \cdot (-1)}{3^2 + 1} = \dfrac{-2}{10} = -\dfrac{1}{5} = -0.20$

If $x = 1$, $y = 3$, then $\dfrac{2 \cdot 3}{1^2 + 1} = \dfrac{6}{2} = 3$.

But $\dfrac{2y}{x^2+1}$ cannot be simplified further.

Problem We have the rational expression

$$\frac{x^2 - x - 6}{x - 3}$$

a) Fill in

x	1	5	-2	2	8	3
$\frac{x^2-x-6}{x-3}$	3	7	0	4	10	" $\frac{0}{0}$ "

'undefined !

b) Find the pattern.

- Add 2 to the x value (except $x=3$)

shorter: $2+x \quad (x \neq 3)$

Quadratic expansion:

$$(x+r)^2 = x^2 + 2rx + r^2$$

Ex: $(x+5)^2 = x^2 + 10x + 25$

Ex: $13^2 = (10+3)^2 = 10^2 + 6\cdot 10 + 9$
$= 169$

Conjungate expansion:

$$(x-r)(x+r) = x^2 - r^2$$

Ex: $(x-5)(x+5) = x^2 - 25$

Ex: $8 \cdot 12 = (10-2)(10+2) = 10^2 - 2^2 = 96$

3. Roots

The square root of 5 is the positive number a such that $a \cdot a = 5$.

(a is in the calculator $a = 2.2361\ldots$)

We write a as $\sqrt{5}$

Note: Negative numbers don't have square roots.

$\sqrt{0} = 0$

Problem Compute (without calc.)

a) $(\sqrt{2}+3)^2 = (\sqrt{2})^2 + 2\sqrt{2} \cdot 3 + 3^2 = \underline{\underline{11 + 6\sqrt{2}}}$

b) $(\sqrt{5}-1)(\sqrt{5}+1) = (\sqrt{5})^2 - 1^2 = 5 - 1 = \underline{\underline{4}}$

There are other roots:

$\sqrt[3]{5}$ is the number a such that $a \cdot a \cdot a = 5$

$\sqrt[5]{32} = 2$ (since $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$)

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4. Powers — repeated multiplication

Ex: $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$ "three to the power of four"

$4 \cdot 4 \cdot 4 = 4^3$

exponent

$4^3 \neq 4 \cdot 3$

base

$4^3 = 64$, $4 \cdot 3 = 12$

$$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$$
$$= 10^5$$
$$= 10^{2+3}$$

So $\quad a^n \cdot a^m = a^{n+m}$

$$\frac{3^6}{3^4} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{3 \cdot 3}{1} = 3^2$$
$$= 3^{6-4} \qquad \left(\text{so } 3^{-4} = \frac{1}{3^4}\right)$$

$$1 = \frac{5^3}{5^3} = 5^{3-3} = 5^0$$

$$(a^n)^m = a^{n \cdot m}$$

Ex: $(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$
$$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$
$$= 3^8$$
$$= 3^{2 \cdot 4}$$

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5. Order of operations

Problem: Compute

a) $2 + 3 \cdot 4 \overset{?}{=} \begin{cases} 5 \cdot 4 = 20 & = (2+3) \cdot 4 \\ 2 + 12 = 14 \end{cases}$

b) $2 \cdot 2^2 \overset{?}{=} \begin{cases} 2 \cdot 4 = 8 \\ 4^2 = 16 & = (2 \cdot 2)^2 \end{cases}$

<u>Problem</u> $-5^2 \overset{?}{=} \begin{cases} 25 = (-5)^2 \\ -25 \end{cases}$ $\qquad -x^2$

$-3x^2$

$$-5^2 = (-1) \cdot 5^2 = -25$$

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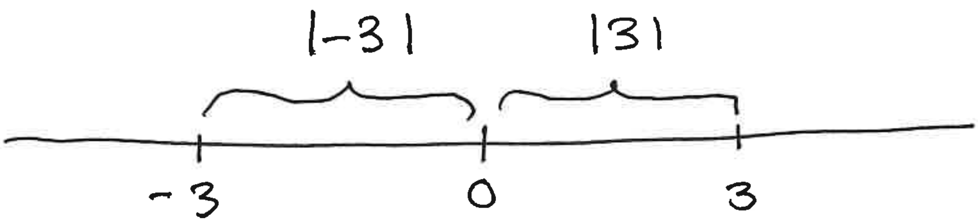
6. Absolute value

If a is a number, then $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

"the absolute value of a"

<u>Ex</u> $|3| = 3$ and $|-3| = -(-3) = 3$

$|a|$ = distance between 0 and a on the number line.



<u>Problem</u> Simplify $\sqrt{x^2}$

Solution: If $x \geq 0$, then $\sqrt{x^2} = x$

If $x < 0$, then $\sqrt{x^2} = -x$

In short: $\sqrt{x^2} = |x|$

<u>Ex</u>: $\sqrt{(x-5)^2} = |x-5|$

<u>Ex</u>: $\sqrt{(-3)^2} = \sqrt{9} = 3 = |-3|$