EBA 2911, lecture 3, 28 Aug. 2019, Runar Ile

Plan: 1. Repetion
2. Total present value
3. Finite geometric series
4. Annuities.

1. Repetition

$$\text{Relative change} = \frac{\text{new value} - \text{old value}}{\text{old value}}$$

$$\text{Rate of change} = 1 + \text{relative change}$$

Ex: The value of Kåre's flat increases by 10% the first year and decreases by 30% the second year. Compute the relative change for the two years combined.

Solution: We put $r_1 = 0.1$ and $r_2 = -0.3$

Rate of change for year 1: $1 + r_1 = 1.1$

Rate of change for y. 2: $1 + r_2 = 0,7$

Rate of change for the two years combined is then

$$(1+r_1)\cdot(1+r_2) = 1.1 \cdot 0.7 = 0.77$$

The relative change for the two years comb. is

$$0.77 - 1 = \underline{\underline{-0.23}}.$$

Pattern: Relative changes of value: $r_1, r_2, \dots, r_n$ give a combined relative change

$$\underbrace{(1+r_1)\cdot(1+r_2)\cdot(1+r_3)\cdot\ldots\cdot(1+r_n)}_{\text{combined rate of change}} - 1$$

Calculate interest: Deposit (principal) $B_0 = 50\,000$ and interest $r = 4\%$. After 5 years the balance is $50\,000 \cdot 1.04^5 = \underline{\underline{60\,832.65}}$

$$(= 50\,000 \cdot (1+4\%)^5)$$

Calculator: 50 000 $\boxed{\times}$ 1.04 $\boxed{y^x}$ 5 $\boxed{=}$

Concepts:

compounding: when the interest is added

interest period: the period between compoundings

period rate: the interest per period.

Ex: Deposit: 50 000

Nominal interest: 4 %

Monthly compounding

After 5 years the balance is

$$50\,000 \cdot \left(1 + \underbrace{\frac{4\%}{12}}_{\text{period rate}}\right)^{12\cdot 5} = 50\,000 \cdot \left(1 + \frac{1}{300}\right)^{60}$$

$$= \underline{\underline{61\,049.83}}$$

Effective interest (r_{eff}) = the annual interest which gives the same balance as the periodic rate.

In the ex. $1 + r_{eff} = \left(1 + \frac{4\%}{12}\right)^{12} = 1.040742$

so $r_{eff} = \underline{\underline{4.0742\%}}$

Powers: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ $(a \geq 0)$

Ex: $\sqrt{a} = a^{\frac{1}{2}}$ and $\sqrt[3]{a} = a^{\frac{1}{3}}$ .....

Problem: After 5 years the deposit of 50 000 has become 60 000 (new balance). Calculate the effective interest.

Solution 1: The 5-year growth factor is

$$1 + \frac{60\,000 - 50\,000}{50\,000} = 1.2$$

Then the yearly growth factor is $\sqrt[5]{1.2} = 1.2^{\frac{1}{5}}$ $= 1.2^{0.2} = 1.03714$. So $r_{eff} = \underline{\underline{3.714\%}}$

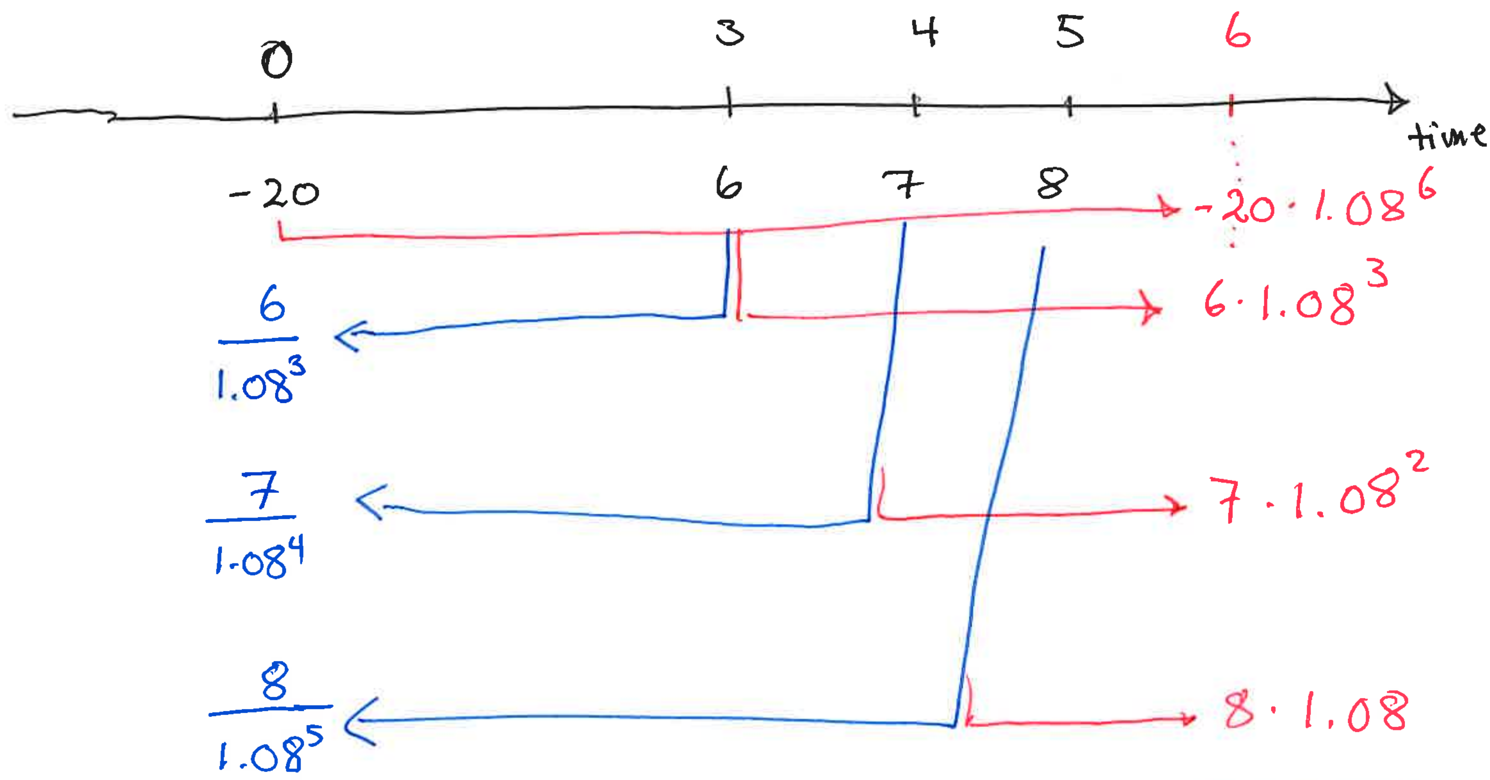
Solution 2:

$$50\,000 \cdot (1 + r_{eff})^5 = 60\,000$$

$$(1 + r_{eff})^5 = \frac{60\,000}{50\,000} = 1.2$$

$$1 + r_{eff} = \sqrt[5]{1.2}$$

$$r_{eff} = \sqrt[5]{1.2} - 1 = \underline{\underline{3.714\%}}$$



The sum is the total present value of the cash flow with 8% interest

$= -4.65$

The sum is the total future value 6 years from now with 8% interest

$= -4.65\cdot 1.08^6$

Internal rate of return: The interest which makes the TPV equal to zero. (in this case 1.1197% – by guesswork)

Present value of payment (K) n years (periods) from now with interest r

= what you have to deposit today (K_0) to get K n years from now if the interest is r.

Since $$K = K_0 \cdot (1+r)^n$$

so $$K_0 = \frac{K}{(1+r)^n} \quad \left(= K \cdot (1+r)^{-n}\right)$$

Ex: 50 000 (K) 3 years from now with interest 4% has present value

$$K_0 = \frac{50\,000}{1.04^3} = \underline{\underline{44\,449.82}}$$

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2. Total present value

Ex (cashflow)	now	3 y.	4 y.	5 y.
	-20	6	7	8

tot. Present value: (8% interest)

$$-20 + \frac{6}{1.08^3} + \frac{7}{1.08^4} + \frac{8}{1.08^5}$$

Problem: Investing 120 mill. is supposed to give payments of 70 mill. 4 years from now and 90 mill. 6 years from now.

The discount rate (interest) is put equal to 12%.

a) Calculate the total present value of the cash flow.

b) Do you think this is a good investment?

Solution.

a)

$$\frac{70}{1.12^4}$$

$$\frac{90}{1.12^6}$$

The sum = tot. pres. value

$= \underline{\underline{-29.92}}$.

b) You will not get 12% rate of return on this investment. But if you pay 90 mill (instead of 120) then the proposal would be fair.

In this problem the internal rate of return is (approx.) 5.8 % because

$$-120 + \frac{70}{1.058^4} + \frac{90}{1.058^6} = 0,0 \text{ (approx)}$$

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3. Finite Series — long sums

Ex: $1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{100}$ is a series with 10 terms.

– a term

We write $a_1 + a_2 + a_3 + \dots + a_{10}$

third term

the 10^{th} term

In ex: $a_1 = 1$, $a_2 = \frac{1}{4}$ ....

Geometric series $a_1 + a_2 + \dots + a_n$

– each term is the previous term multiplied with a constant k.

so $a_2 = k \cdot a_1$, $a_3 = k \cdot a_2$, $a_4 = k \cdot a_3$ ....

$$a_3 = k \cdot k \cdot a_1 = k^2 a_1 \qquad a_4 = k \cdot k^2 a_1 = k^3 \cdot a_1$$

We can find an expression for the sum

$$a_1 + a_2 + \dots + a_n = a_1 + k \cdot a_1 + k^2 \cdot a_1 + \dots + k^{n-1} \cdot a_1$$

$$= a_1 \underbrace{(1 + k + k^2 + \dots + k^{n-1})}_{\frac{k^n - 1}{k - 1}}$$

Problem: Calculate the sum

$$5 + 5\cdot 1.003 + 5\cdot 1.003^2 + 5\cdot 1.003^3 + \dots + 5\cdot 1.003^{60}$$

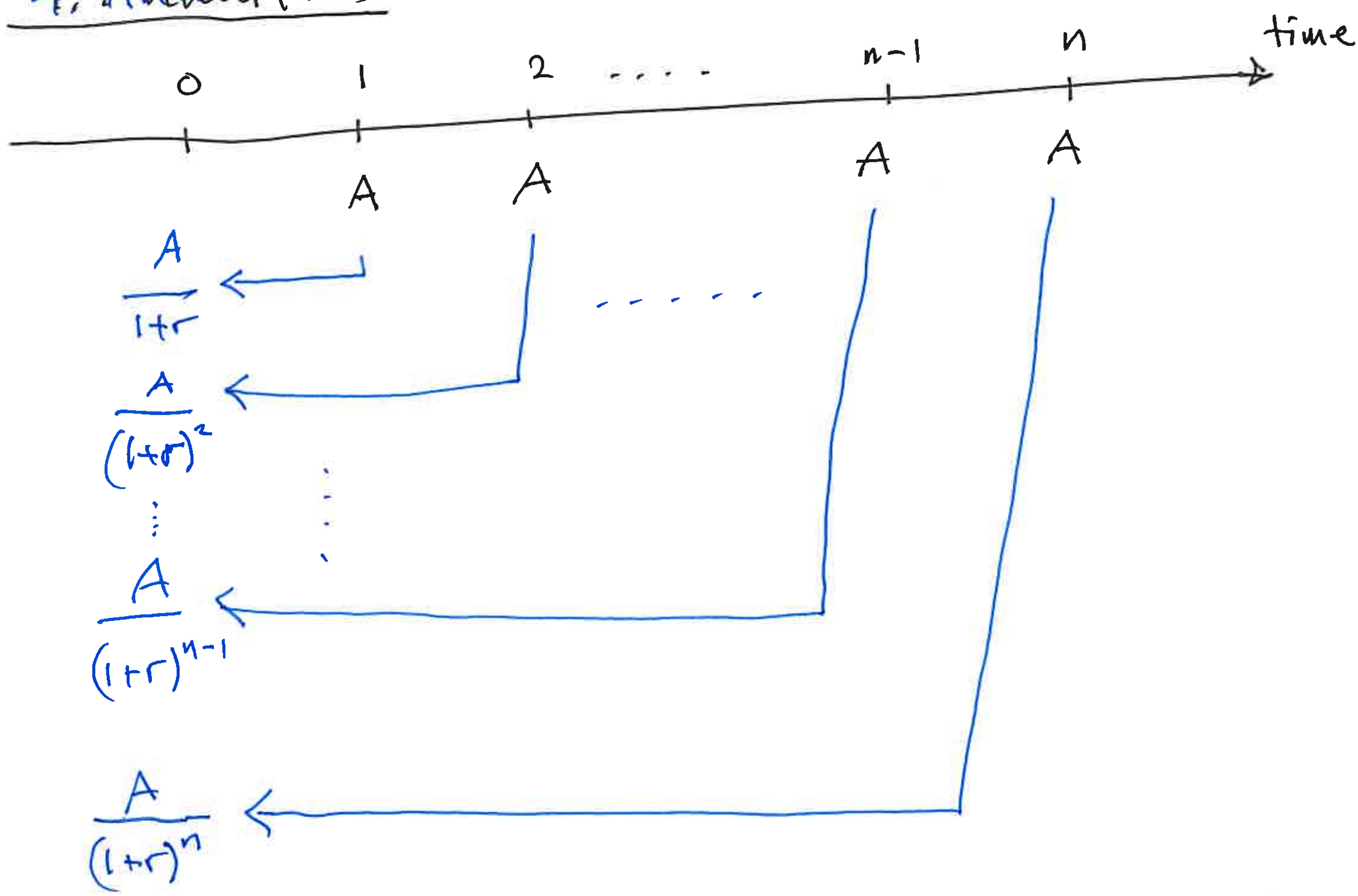
Solution: This is a geometric series

$a_1 = 5$, $k = 1.003$, $n = 61$ so

the sum is $5 \cdot \dfrac{1.003^{61}-1}{1.003-1} = 5 \cdot \dfrac{1.003^{61}-1}{0.003}$

$= \underline{\underline{334.142}}$.

4. Annuities – constant cash flows



The sum (= tot. pres. val.) is geometric series with $a_1 = \dfrac{A}{(1+r)^n}$, $k = 1+r$, n terms

The sum is then

$$\frac{A}{(1+r)^n} + \frac{A}{(1+r)^{n-1}} + \dots + \frac{A}{(1+r)^2} + \frac{A}{1+r} = \frac{A}{(1+r)^n} \cdot \frac{(1+r)^n - 1}{r}$$

$$= A \cdot \frac{(1+r)^n - 1}{r(1+r)^n}$$