EBA 2911, Lecture 1, 19 Aug. 2019, Runar Ile

Plan: 1. Intro. to the course
2. Algebraic expressions
3. Laws of algebra
4. Roots
5. Powers
6. Order of operations

---

1. Intro. to the course

Autumn

- Financial math.
- Functions and graphs
- Differentiation and optimization

Spring

- Integration
- Systems of linear equations
- Functions in two variables $z = f(x, y)$.

---

2. Algebraic expressions

Variables: $x, y, z, x_1, x_2, x_3, \ldots$
$a, b, c, \ldots, m, n, \ldots$

Multiply by a number

short-writing

$$3 \cdot x = 3x = x + x + x$$

$$3 \cdot 2 \neq 32$$

$$\sqrt{3} \cdot x = \sqrt{3}\,x$$

$$(-1) \cdot x = -x$$

$$1 \cdot x = x$$

$$0 \cdot x = 0$$

Addition: $x + x = 2x$

$x + y$ ~~Ex~~ no simplification

$x + y + x = 2x + y$

Multiplication: $x \cdot y = xy$

$x \cdot x = x^2$

$xy \cdot x^2 = x \cdot y \cdot x \cdot x = x^3 y$

Dividing: $\dfrac{x+y}{z}$, $\dfrac{x-y}{x+y}$

<u>Rational</u> <u>expressions</u>: Fractions of polynomials

<u>Ex</u>: $\dfrac{y}{x^2+1}$ ...

Other expressions: $\sqrt{x^2+1}$, $\dfrac{3\sqrt{x}+1}{\sqrt{x}-1}$, ...

– not rational.

We can insert numbers for the variables

<u>Ex</u>: $\dfrac{2y}{x^2+1}$ with $x = 3$, $y = -1$

gives a number: $\dfrac{2 \cdot (-1)}{3^2+1} = \dfrac{-2}{10} = -\dfrac{1}{5}$

$= -0{,}20$

If $x = 1, y = 3$, then we get

$$\frac{2 \cdot 3}{1^2 + 1} = \frac{6}{2} = 3$$

But $\frac{2y}{x^2+1}$ cannot be simplified further

Problem: We have the expression $\frac{x^2 - x - 6}{x - 3}$.

a) Fill in

x	1	5	−2	2	8	3
$\frac{x^2-x-6}{x-3}$	3	7	0	4	10	$\frac{0}{0}$ not a number! – undefine.

b) Find the pattern.

Answer: $x + 2 \quad (x \neq 3)$.

(Reason: $x^2 - x - 6 = (x-3)(x+2)$ so

$$\frac{x^2 - x - 6}{x - 3} = \frac{(x-3)(x+2)}{x-3} \overset{(x \neq 3)}{=} \frac{x+2}{1}$$

$$= x + 2$$

3. Laws of algebra   Suppose a, b, c are expressions (or numbers)

---

(1) Addition is commutative: $a+b=b+a$

Ex: $a = 2x+1$, $b = x-y$, then

$$(2x+1)+(x-y) = (x-y)+(2x+1)$$

---

(2) Multiplication is commutative: $a \cdot b = b \cdot a$

Ex: $(2x+1)\cdot(x-y) = (x-y)\cdot(2x+1)$

---

(3) Addition is associative:

$$(a+b)+c = a+(b+c)$$

Ex: $(2x+3)+y = 2x+(3+y)$

---

(4) Multiplication is associative:

$$(a \cdot b)\cdot c = a \cdot (b \cdot c)$$

Ex: $(3 \cdot x)\cdot y = 3 \cdot (x \cdot y)$

Ex: $2 \cdot (3 \cdot 4)$ $= 2 \cdot 12$ $= 24$; $(2 \cdot 3)\cdot 4$ $= 6 \cdot 4$ $= 24$

(5) The distributive law

$$a\cdot(b+c) = a\cdot b + a\cdot c$$

Ex: $2\cdot(3+4)$ $\qquad$ $2\cdot 3 + 2\cdot 4$

$= 2\cdot 7$ $\qquad$ $= 6 + 8$

$= 14$ $\qquad$ $= 14$

Ex: $x(x+3) = x\cdot x + x\cdot 3 = x^2 + 3x$

Quadratic expansion

$$(x+r)^2 = \underbrace{(x+r)}_{a}(\underbrace{x}_{b}+\underbrace{r}_{c})$$

$$\overset{\text{distr. law}}{=} (x+r)\cdot x + (x+r)\cdot r$$

$$\overset{\text{mult. is comm.}}{=} x(x+r) + r(x+r)$$

$$\overset{\text{distr. l.}}{=} x^2 + x\cdot r + rx + r^2$$

$$\overset{\text{mult. is com.}}{=} x^2 + rx + rx + r^2$$

$$\overset{\text{distr. law}}{=} x^2 + 2rx + r^2$$

always the same answer

Ex: $(x+1)^2 \overset{r=1}{=} x^2 + 2\cdot 1\cdot x + 1^2 = x^2 + 2x + 1$

Problem: Use quadratic expansion to write $x^2 + 6x + 9$ as a square.

Solution: $x^2 + 2rx + r^2$, to be equal

we need $\begin{cases} 2r = 6 & \text{so} \quad r = \frac{6}{2} = 3 \\ r^2 = 9 & \text{and} \quad \underset{\text{lhs}}{3^2} = \underset{\text{rhs}}{9} \text{ is ok} \end{cases}$

By the quad. expansion result

$$x^2 + 6x + 9 = (x+3)^2$$

Conjugate expansion

$$(x-r)(x+r) \overset{\text{distr. law}}{=} (x-r)\cdot x + (x-r)\cdot r$$

$$\overset{\text{distr law}}{=} x^2 - rx + rx - r^2$$

$$= x^2 - r^2$$

always the same answer

Ex: $99 = 100 - 1 = 10^2 - 1^2 = (10-1)(10+1)$

$= 11 \cdot 9$

Ex: $600 = 20 \cdot 30 = (25-5)(25+5) = 25^2 - 5^2$

so $25^2 = 600 + 25 = 625$

<u>Problem</u>: Use the quadratic and conjugate expansion to factorise the expressions.

a) $x^2 - 16$ $\quad (x-4)(x+4)$ $\quad (r=4)$

b) $x^2 - 2x + 1$ $\quad (x-1)^2$ $\quad (r=-1)$

c) $x^2 + 10x + 25$ $\quad (x+5)^2$ $\quad (r=5)$

d) $9x^2 - 4y^2$ $\quad (3x-2y)(3x+2y)$ $\quad (r=2y)$

e) $x^2 + 6xy + 9y^2$ $\quad (x+3y)^2$ $\quad (r=3y)$

$$x^2 + 2rx + r^2$$

(e): $x^2 + (6y)x + (9y^2)$ so $\begin{cases} 2r = 6y \Rightarrow r = 3y \\ r^2 = 9y^2 \text{ - is ok} \end{cases}$

<u>4. Roots</u>

The square root of 5 is the positive number a such that $a \cdot a = 5$.

(a is in the calculator $a = 2.2361\ldots$

We write $a = \sqrt{5}$

Note: Negative numbers don't have square roots!

$\sqrt{0} = 0$

Problem: Compute (without calc.)

a) $(\sqrt{2}+3)^2$

Use quadratic expansion

$(\sqrt{2})^2 + 2\sqrt{2}\cdot 3 + 3^2$

$= 2 + 6\sqrt{2} + 9 = 11 + 6\sqrt{2}$

b) $(\sqrt{5}-1)(\sqrt{5}+1)$ Use conjugate expansion

$(\sqrt{5})^2 - 1^2 = 5 - 1 = 4$

There are other roots:

$\sqrt[3]{5}$ is the number a such that $a \cdot a \cdot a = 5$

(and $a = 1.7100\ldots$)

$\sqrt[5]{32} = 2$

---

5. Powers – repeated multiplication

Ex: $3 \cdot 3 \cdot 3 \cdot 3 \overset{\text{short}}{=} 3^4$ "three to the power of four"

$4 \cdot 4 \cdot 4 = 4^3$

exponent

$4^3 \neq 4 \cdot 3$

base

$4^3 = 64$, $4 \cdot 3 = 12$

$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$

$= 10^5$

$= 10^{2+3}$ so $a^n \cdot a^m = a^{n+m}$

$$\frac{3^6}{3^4} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{3 \cdot 3}{1} = 3^2$$

$$= 3^{6-4} \qquad \left(\text{so } \; 3^{-4} = \frac{1}{3^4}\right)$$

$$1 = \frac{5^3}{5^3} = 5^{3-3} = 5^0$$

$$(a^n)^m = a^{n \cdot m}$$

Ex: $(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$

$$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

$$= 3^8$$

$$= 3^{2 \cdot 4}$$

---

6. Order of operations

$(2+3) \cdot 4$

Problem: compute

a) $2 + 3 \cdot 4 = \begin{cases} 5 \cdot 4 = 20 \\ 2 + 12 = 14 \end{cases}$ write

b) $2 \cdot 2^2 = \begin{cases} 4^2 = 16 \\ 2 \cdot 4 = 8 \end{cases}$ $\xrightarrow{\text{write}} (2 \cdot 2)^2$

Homework: Press $2 \boxed{+} 3 \boxed{\times} 4$ on your BI-calc.