Course paper	EBA 29103 Mathematics for Business Analytics
Deadline	November 14th 2022 at 1200

The problem set consists of two pages. All subquestion have equal weight, and at least 60% score is required to pass. **You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.** Your answers should be provided as a single file in PDF format.

Question 1.
Compute the integrals:

a) $\int 15\sqrt{x}\ \mathrm{d}x$ b) $\int \frac{2}{x^2}\ \mathrm{d}x$ c) $\int 2x(1-6x^2)\ \mathrm{d}x$ d) $\int 12(1-x)^5\ \mathrm{d}x$

Question 2.
Compute the integrals:

a) $\int \frac{e^x}{1-e^x}\ \mathrm{d}x$ b) $\int \frac{1-x}{1-4x^2}\ \mathrm{d}x$ c) $\int \frac{3(\ln x)^2}{x}\ \mathrm{d}x$ d) $\int 6x^2\, e^{-x\sqrt{x}}\ \mathrm{d}x$

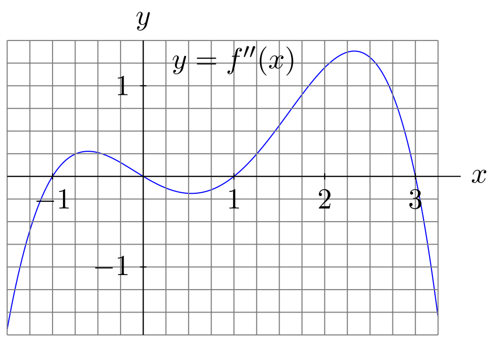
Question 3.
Let E be the ellipse with symmetry lines $x = 2$ and $y = 1$ going through the points (5,1) and (2,3), and let H be the hyperbola going through the point (2,3) with $x = -1$ and $y = -1$ as asymptotes.

a) Find the equation of the ellipse E and the hyperbola H.
b) Make a figure showing E, H, and the area S bounded by E, H and $x = 2$, and compute the area of S. You may use that the area of an ellipse with half-axes $a,b > 0$ is given by πab.

Question 4.
Let $f(x)$ be the net cash flow after x years (in million NOK per year) from a rental property. We think of this as a continuous cash flow, and use continuous discounting with discount rate $r = 10\%$ to compute net present values. Find the total net present value from the rental property in the first 10 years when

a) $f(x) = 100 + 4x$ b) $f(x) = 100 \cdot 1.04^x$

Question 5.
The graph of $f''(x)$ is shown in the figure below. Use the figure to estimate the value of the integral $\int_0^3 f''(x)\ \mathrm{d}x$. What can you say about $f'(0)$ and $f'(3)$?



Question 6.
Use Gaussian elimination to solve the linear systems. Show the elementary row operations, mark the pivot positions in the echelon form, and specify the number of solutions:

$$a)\quad \begin{array}{rcrcrcr} x & + & 2y & - & z & = & 3 \\ 5x & + & 8y & - & 2z & = & 23 \\ 2x & + & 6y & - & 5z & = & 6 \\ 6x & + & 10y & - & 3z & = & 27 \end{array} \qquad b)\quad \begin{array}{rcrcrcrcr} x & + & 2y & + & 4z & + & w & = & 11 \\ 2x & + & 5y & + & 4z & - & 3w & = & 18 \\ 2x & + & 3y & + & 8z & + & 3w & = & 10 \end{array}$$

Question 7.
Compute the determinant $|A|$, and determine when $|A| = 0$:

$$\text{a) } A = \begin{pmatrix} 1 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & a & 7 \end{pmatrix} \qquad \text{b) } A = \begin{pmatrix} s & s & 2 \\ s & -s & 0 \\ 1 & 1 & s \end{pmatrix} \qquad \text{c) } A = \begin{pmatrix} 1 & t & 0 & 0 \\ t & 2 & 0 & 0 \\ 0 & 0 & t & 1 \\ 0 & 0 & 8 & t \end{pmatrix}$$

Question 8.
Let $\mathbf{v}_1 = (1,3,2,4)$, $\mathbf{v}_2 = (2,5,6,7)$, og $\mathbf{v}_3 = (3,6,-2,2)$.

a) Determine whether the vector $\mathbf{w} = (1,1,4,1)$ is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.
b) Determine all vectors that are linear combinations of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

Question 9.
The linear system $A\mathbf{x} = \mathbf{b}$ is given by

$$A = \begin{pmatrix} 2 & 5 & 3 \\ 3 & 7 & a \\ 5 & a & 35 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 4 \\ -8 \\ -144 \end{pmatrix}$$

where a is a parameter.

a) Find A^{-1} when $a = 0$.
b) Determine all values of a such that $A\mathbf{x} = \mathbf{b}$ has a unique solution.
c) Find all solutions of $A\mathbf{x} = \mathbf{b}$ in the cases where there are infinitely many solutions.
d) Find the z-coordinate of the solution (x,y,z) in the cases where $A\mathbf{x} = \mathbf{b}$ has a unique solution.