EVALUATION GUIDELINES - Multiple choice

EBA 29102
Mathematics for Business Analytics

Department of Economics

Start date:	11.12.2019	Time 09:00
Finish date:	11.12.2019	Time 12:00

For more information about formalities, see examination paper.

Multiple choice 1 in EBA2910[1] - Mathematics for Business Analytics

11 December 2019

SOLUTIONS

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Correct answer	B	C	C	A	A	B	D	B	D	D	A	B	D	C	A

Problem 1

The polynomial division

$$
\begin{array}{l}
\big(\ 2x^3 \qquad\quad -5x \ \ +3\big) : \big(x-3\big) = 2x^2+6x+13+\dfrac{42}{x-3} \\
\underline{-2x^3+6x^2} \\
\qquad\quad 6x^2 \ -5x \\
\qquad\ \underline{-6x^2+18x} \\
\qquad\qquad\quad 13x \ +3 \\
\qquad\qquad\ \underline{-13x+39} \\
\qquad\qquad\qquad\quad 42
\end{array}
$$

gives 42 as remainder (B).

Problem 2

(A) $f'(x) = e^x$ hence $f'(-1) = e^{-1} = 0.3679 < 0.37$.
(B) By the power rule $f'(x) = \left(x^{0.5}\right)' = 0.5x^{-0.5} = \frac{1}{2\sqrt{x}}$ hence $f'(4) = \frac{1}{2\sqrt{4}} = 0.25$.
(C) By the quotient rule

$$f'(x) = \left(\frac{x-1}{2x-3}\right)' = \frac{1\cdot(2x-3)-(x-1)\cdot 2}{(2x-3)^2} = \frac{2x-3-2x+2}{(2x-3)^2} = -\frac{1}{(2x-3)^2}$$

hence $f'(1) = -1$ (and not -5).
(D) By the product rule $f'(x) = (x\ln(x))' = \ln(x)+1$ hence $f'(1) = 1$.
Hence (C) is wrong.

Problem 3

Becuase $e^{0.1x}$ is positive for all x we can divide each side of the equation by $e^{0.1x}$ and get the equivalent equation $x^2 - 9 = 0$ which gives $x = \pm 3$. (C) is correct.

Problem 4

The function $f(x)$ has per definition stationary points for those x where $f'(x) = 0$, i.e. where the tangent of the graph of $f(x)$ is horizontal. There are 4 such points on this graph. Hence (A) is wrong.

Problem 5

Because $f(x)$ is continuous on the closed interval, $f(x)$ has a least one global maximum point and one global minimum point. The extremal points of $f(x)$ are either stationary points or end points (there are no cusps). We have $f'(x) = -3x^2 + 12x - 9$. This is a concave second degree function with zeros $x = 1$, $x = 3$, a local minimum point and a local maximum point, respectively. (We can also use the second derivative test: $f''(1) = 6 > 0$, $f''(3) = -6 < 0$.) We compute $f(0) = 10$, $f(1) = 6$, $f(3) = 10$ and $f(4) = 6$. $f(x)$ has two local minimum points $x = 1$ and $x = 4$, and so (A).

[1] Exam code EBA29102

Problem 6

The monthly interest rate is $3.6\% : 12 = 0.003$. The period before the first payment equals 48 terms, hence the present value of the first payment is $\frac{12\,000}{1.003^{48}}$. Payments for 30 years give 360 terms. The present value of the last payment is then $\dfrac{12\,000}{1.003^{407}}$. This gives (B).

Problem 7

The present value of the cash flow should be 0 which gives the equation $-30+\frac{50}{e^{4r}}=0$, i.e. $\frac{50}{e^{4r}}=30$, i.e. $e^{4r}=\frac{5}{3}$. Then we put the left hand side and the right hand side into the strictly increasing function $\ln(x)$ and get the equivalent equation $4r=\ln(\frac{5}{3})=\ln(5)-\ln(3)$, i.e. $r=\frac{\ln(5)-\ln(3)}{4}$ which is (D).

Problem 8

$\ln(x-1)$ is only defined for $x>1$. We put the left hand side and the right hand side into the strictly increasing function e^x. This gives the equivalent inequality $x-1\leqslant e^2$ with the extra condition $x>1$. The solution is $x\in\langle 1\,,\,1+e^2]$ which is (B).

Problem 9

From the graph we see the symmetry axis of the parabola $x=9$ and the maximum value 20. Hence $f(x)=a(x-9)^2+20$ for some undetermined number a. We note that $f(14)=15$, i.e. $a\cdot 5^2+20=15$, i.e. $a=-0.2$. Hence $f(x)=-0.2(x-9)^2+20$ and $f(0)=-0.2(-9)^2+20=3.8$ which gives (D).

Problem 10

We calculate the limits by l'Hôpital's rule.

(A) $\displaystyle\lim_{x\to\infty}\frac{2x+1}{3x-7}\overset{\text{l'Hôp}}{=}\frac{2}{3}$.

(B) $\displaystyle\lim_{x\to\infty}\frac{\ln(3x^2+1)}{x}\overset{\text{l'Hôp}}{=}\lim_{x\to\infty}\frac{\frac{6x}{3x^2+1}}{1}=\lim_{x\to\infty}\frac{6x}{3x^2+1}\overset{\text{l'Hôp}}{=}\lim_{x\to\infty}\frac{6}{6x}=0.$

(C) $\displaystyle\lim_{x\to\infty}\frac{x}{e^x+1}\overset{\text{l'Hôp}}{=}\lim_{x\to\infty}\frac{1}{e^x}=0.$

(D) $\displaystyle\lim_{x\to\infty}\frac{(1-3x)(4x+1)}{(2x-7)(1-2x)}=\lim_{x\to\infty}\frac{-12x^2+x+1}{-4x^2+16x-7}\overset{\text{l'Hôp}}{=}\lim_{x\to\infty}\frac{-24x+1}{-8x+16}\overset{\text{l'Hôp}}{=}\frac{-24}{-8}=3$ which is the only answer in the interval $[2\,,\,5]$.

Problem 11

We calculate $D'(p)=-3$ and the elasticity function is hence $\varepsilon(p)=\frac{-3p}{60-3p}$. The demand is elastic with respect to price if $\varepsilon(p)<-1$. It gives the equivalent inequality $\frac{60-6p}{60-3p}<0$. Since $60-3p$ is positive for all p in the interval $\langle 0\,,\,20\rangle$ the sign of the numerator determines the sign of the quotient. We get $60-6p<0$ which is true for all p in the interval $\langle 10\,,\,20\rangle$. So (A).

Problem 12

In figure 1 we see the tangents (k_1–k_4) of the graphs K_1–K_4 containing the origin. The slope of the tangent gives optimal (minimal) average unit cost. We see that k_2 (the red) has the smallest slope. This gives (B).

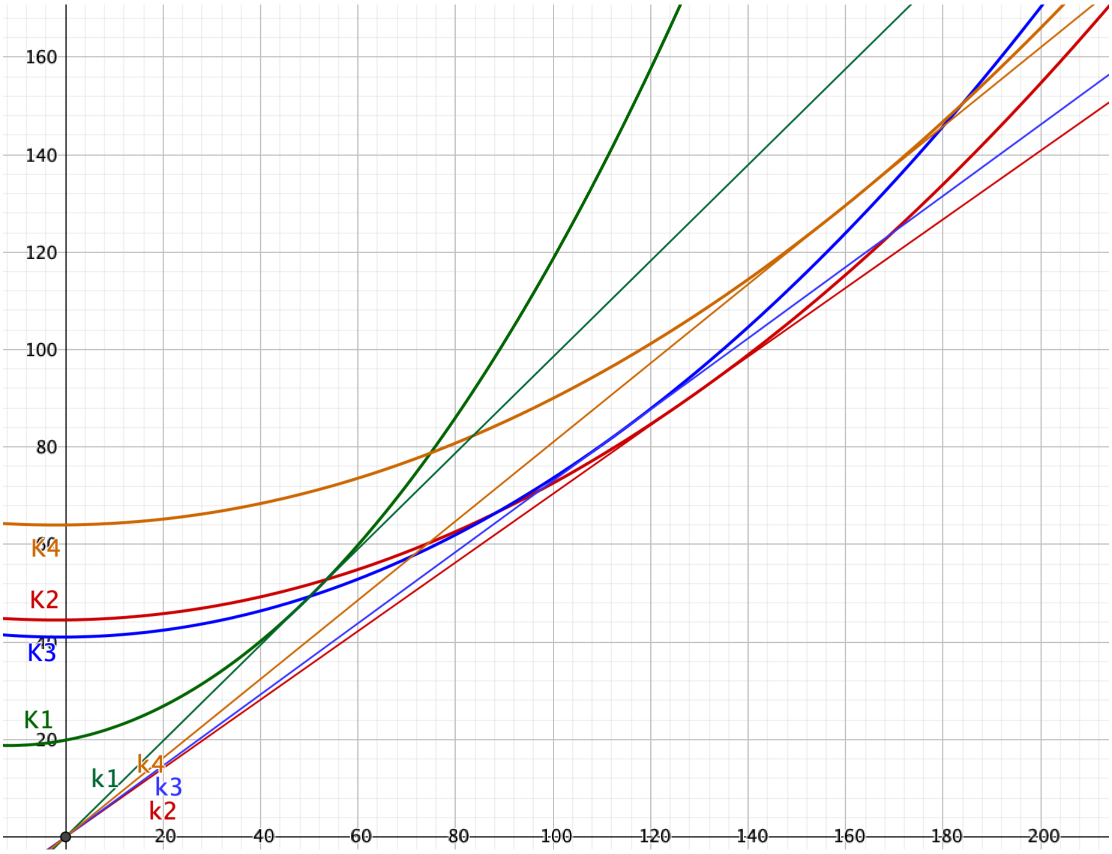

Figure 1: Four cost functions (K_1–K_4) with tangents (k_1–k_4)

Problem 13

We want to find the standard form of the ellipse equation. We collect terms with x or y on the left hand side: $16x^2 - 96x + 9y^2 - 72y = -144$. We want to complete the squares and factors 16 and 9 outside: $16[x^2 - 6x] + 9[y^2 - 8y] = -144$. Now we can complete the squares on the inside of the parantheses: $16[(x-3)^2 - 9] + 9[(y-4)^2 - 16] = -144$. Then we multiply back into the outer parantheses: $16(x-3)^2 - 144 + 9(y-4)^2 - 144 = -144$, that is $16(x-3)^2 + 9(y-4)^2 = 144$. Finally we divide by 144 to get the standard form:

$$\frac{(x-3)^2}{9} + \frac{(y-4)^2}{16} = 144$$

The ellipse hence has centre $(3, 4)$ and half axes 3 and 4. This is only compatible with (D).

Problem 14

(A) $f'(x)$ has a minimum point $x \approx 2.65$ and a maximum point $x \approx 4.35$. These are the only points where $f''(x)$ changes sign and thus are the only inflection points for $f(x)$.
(B) From the graph we see that the slope of the tangent of $f'(x)$ (i.e. $f''(x)$) is increasing from $x = 3$ to $x \approx 3.5$ and decreasing from $x \approx 3.5$ to $x = 4$.
(C) Since $f'(x)$ is negative for x in the interval $\langle 2, 3\rangle$, $f(x)$ is strictly decreasing in the interval $[2, 3]$ and hence $f(2) > f(3)$ is correct.
(D) $f(x)$ is concave where $f''(x)$ is less or equal 0, but from the graph we see that the slope of the tangent of $f'(x)$ is positive from $x = 4$ to $x \approx 4.35$.

Problem 15

We denote the equation $e^{y^2+x} = 8x^2$ by $(*)$. We differentiate both sides of $(*)$ to get an equation which contains y'. By applying the chain rule twice we get $(2yy' + 1)e^{y^2+x} = 16x$. We want to solve this for y'. We substitute $e^{y^2+x} = 8x^2$ and get $(2yy' + 1) \cdot 8x^2 = 16x$. By dividing with $8x^2$ on each side (ok because $x = 0$ doesn't give any solutions to $(*)$) we get $2yy' + 1 = \frac{2}{x}$, i.e. $2yy' = \frac{2-x}{x}$ which gives $y' = \frac{2-x}{2xy}$. We see that $x = 2$ implies $y' = 0$. Here we use that the corresponding y-values are different from 0. We find the y-values by inserting $x = 2$ into $(*)$ and solving, i.e. solving $e^{y^2+2} = 32$. We put the left hand side and the right hand side into $\ln(x)$ abd get the equation $y^2 + 2 = \ln(32)$, i.e. $y^2 = \ln(32) - 2$. Because $\ln(32) - 2 > 0$, this equation has exactly two solutions for y (and none of them equals 0). This gives (A).

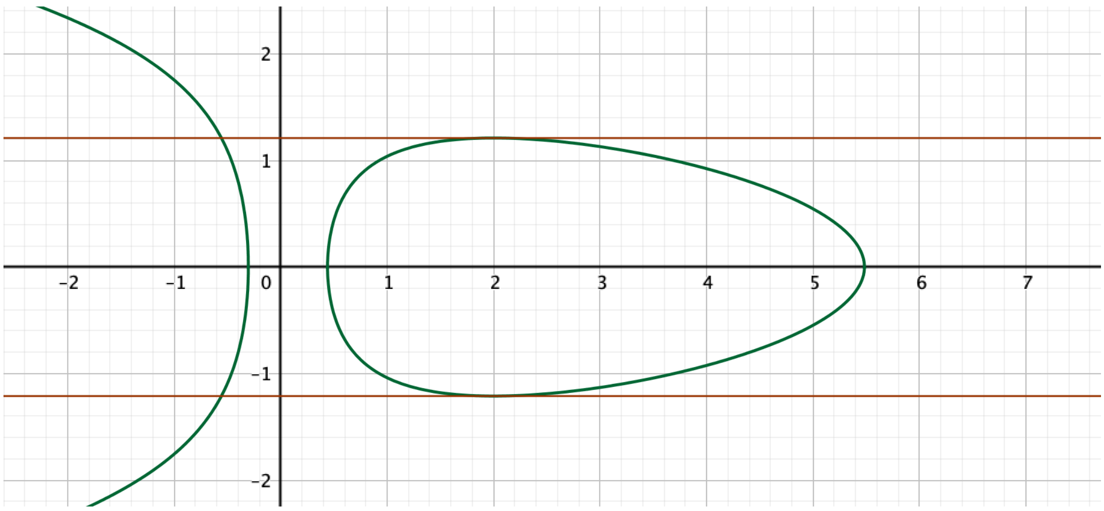

Figure 2: The implicitly defined curve (green) with tangents (brown) for $x = 2$