

School exam (3h) EBA11805 - Mathematics for Data Science

6 December 2024

The exam set has 3 pages. All 12 problems have equal weight. You are required to give reasons for all answers. Grades: A - F which counts for 40% of the final grade in the course.

Support materials permitted: BI-approved exam calculator. Ruler.

Problem 1

We have the function $f(x) = e^{0.5x^2 - 3x}$.

- Determine the derivative function $f'(x)$.
- Determine the stationary points of $f(x)$ and determine where $f(x)$ is increasing and decreasing.

Problem 2

Make a drawing of the ellipse given by the equation $x^2 + 9(y - 5)^2 = 36$.

Problem 3

We have a cost function $K(x)$ with graph as in figure 1. Use the graph to find cost optimum and minimal average unit cost.

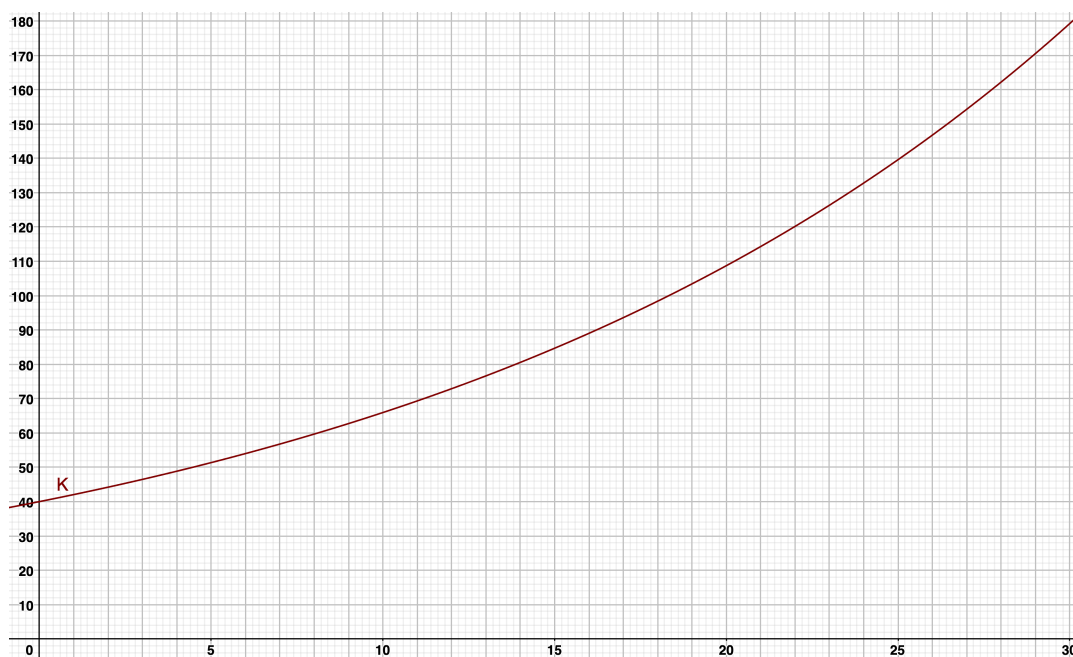


Figure 1: Cost function

Problem 4

- Calculate the sum of the geometric series

$$8000 \cdot 1.005^{143} + 8000 \cdot 1.005^{142} + 8000 \cdot 1.005^{141} + \dots + 8000 \cdot 1.005^{25} + 8000 \cdot 1.005^{24}.$$

- Describe a finance situation where this sum is relevant (the important numbers should be interpreted).

Problem 5

The hyperbola function $f(x)$ has graph as in figure 2. Determine the expression of the hyperbola function.

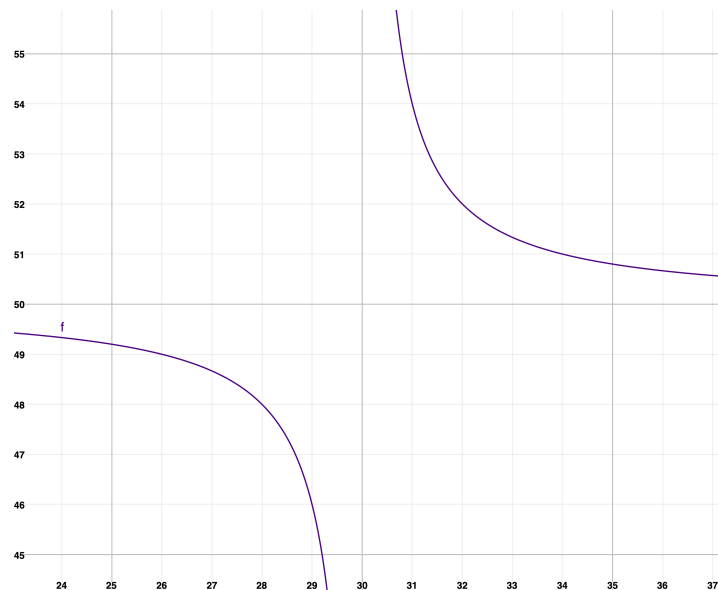


Figure 2: Hyperbola function

Problem 6

Determine the expression of the second degree polynomial function $f(x)$ if the graph passes through the point $P = (10, 20)$ and has maximum point $Q = (12, 24)$.

Problem 7

Draw a sketch of the graph of a function $f(x)$ which is strictly decreasing, has $f(10) = 20$, $f''(x) < 0$ for all x between 5 and 15, and $f''(x) > 0$ for all $x > 15$. (Note: you are not supposed find any function expressions!)

Problem 8

Let p be the price of a commodity and assume $D(p) = -\ln(p)$ with $0 < p < 1$ is the demand function. Let $\varepsilon(p)$ be the momentary price elasticity of the demand function.

- i) Calculate $\varepsilon(p)$.
- ii) Determine the price p such that the demand is elastic.
- iii) Determine whether the revenue is increasing or decreasing if the price increases a little from $p = 0.5$.

Problem 9

The second derivative of the function $f(x)$ is given as the following function:

$$f''(x) = (e^x - 7)\ln(x^2 + 0.5)$$

- i) Determine the inflection points of $f(x)$.
- ii) Determine where $f(x)$ is convex and concave.

Problem 10

Here are some payments at different times:

Year	3	4	5
Payment	A	B	C

Assume the nominal discount rate is r with continuous compounding.

- i) Write up an expression for the present value of the cash flow.
- ii) Assume $A = 40$, $B = 60$, $C = 100$. Determine r such that the present value of payment C is greater than the present value of payment B .

Problem 11

We have the function $f(x) = \frac{60e^{-0.01x} + 100}{e^{-0.01x} + 4}$ with domain of definition $D_f = [0, \rightarrow)$. Let $g(x)$ be the inverse function of $f(x)$.

- i) Determine the expression of $g(x)$.
- ii) Determine the domain of definition D_g and the range R_g of $g(x)$.

Problem 12

We have the function $f(x) = \frac{x^3 + 7x^2 - 100x - 700}{x^2 + 2x - 35}$. Determine possible maximum and minimum on the interval $\langle 5, \rightarrow)$.