

## School exam (3h) EBA11805 - Mathematics for Data Science

16 May 2024

The exam set has 3 pages. All 12 problems have equal weight. You are required to give reasons for all answers. Grades: A - F which counts for 40% of the final grade in the course.

Support materials permitted: BI-approved exam calculator. Ruler.

### Problem 1

Determine the standard equation for the ellipse in figure 1.

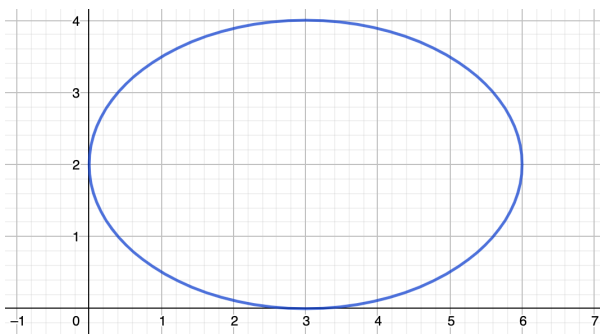


Figure 1: Ellipse

### Problem 2

Determine the function expression  $f(x)$  for the hyperbola in figure 2.

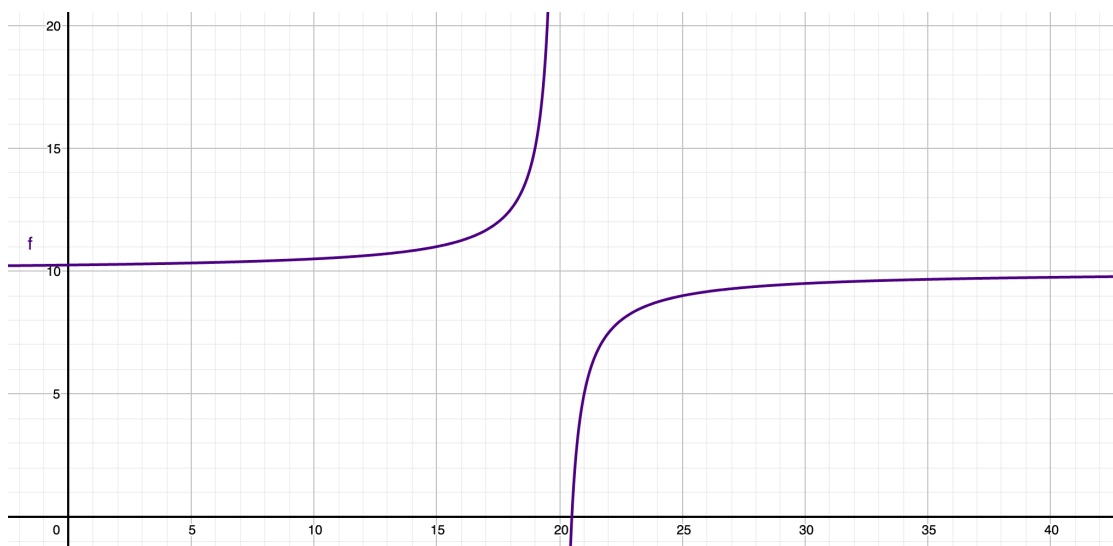


Figure 2: Hyperbola

### Problem 3

Find an expression for the second degree polynomial function  $f(x)$  if the graph has top-point  $P = (6, 10)$  and passes through the point  $Q = (4, 8)$ .

### Problem 4

The graphs of the functions  $f(x)$  and  $g(x)$  are given in figure 3.

- i) Use the figure to solve the inequality  $f(x) \leq g(x)$ .
- ii) Use the figure to solve the inequality  $f(x) \cdot g(x) \geq 0$ .

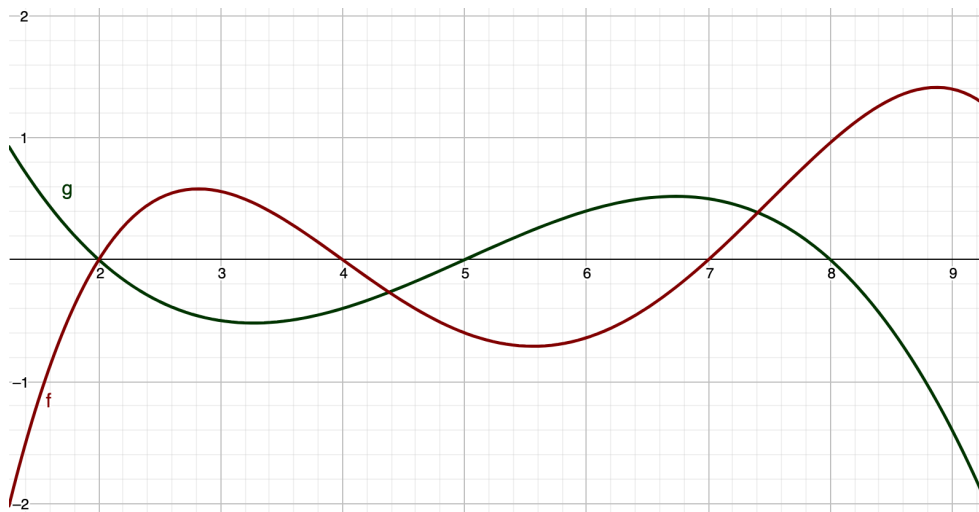


Figure 3: The graphs of  $f(x)$  and  $g(x)$

### Problem 5

The graph of the function  $f(x)$  is given in figure 3.

- i) Use the figure to find an approximation to  $f'(4)$ .
- ii) Use the figure to make the sign-line for  $f''(x)$ .

### Problem 6

We have the function  $f(x) = 30e^{x(10-x)}$ .

- i) Calculate the expression for the derivative function  $f'(x)$  and determine the stationary points for  $f(x)$ .
- ii) Determine the maximal value and minimal value for  $f(x)$  when the domain of definition for  $f(x)$  is  $D_f = [3, 8]$ .

### Problem 7

Kåre is thinking about saving money, with 15 000 each month, with the first deposit 4 years from now. Assume nominal interest is 6% with monthly compounding. Assume last deposit is 12 years from now.

- i) Write up a geometric series for how much Kåre has in his account 12 years from now.
- ii) Calculate how much Kåre has in his account 12 years from now.

**Problem 8**

Here is a cash flow:

Year	0	1	3	5
Payment	-20	-20	25	40

- i) Write up the equation for the internal rate of return (IRR). (Note: you are not supposed to solve the equation!)
- ii) Determine whether the internal rate of return is larger or smaller than 14% (Note: calculation by the finance buttons on the calculator is not a valid argument!).

**Problem 9**

We have the function  $f(x) = 4 + 5e^{-0,1x}$  with domain of definition  $D_f = [0, \rightarrow)$ .

- i) Determine the asymptotes of  $f(x)$ .
- ii) Determine the inverse function  $g(x)$ , determine the domain of definition  $D_g$ , and determine the range  $R_g$ .

**Problem 10**

- i) Solve the equation  $\ln(x^4 - x^2 - 5) = 0$ .
- ii) Use one parameter to write an expression for all polynomials on the form  $x^2 + bx + c$  which have two zeros of distance 6 from each other.

**Problem 11**

- i) Determine the Taylor polynomial  $P_3(x)$  of degree 3 of the function  $f(x) = \ln(x + 1)$  about  $x = 0$ .
- ii) Use  $P_3(x)$  to give an approximate value for  $\ln(1.2)$ .

**Problem 12**

An amount  $K_0$  is deposited in an account today and increases to  $K_{10}$  in 10 years.

- i) Find an expression for the effective annual interest  $r_{\text{eff}}$ .
- ii) Assume there is continuous compounding. Find an expression for the nominal annual interest  $r$ .