School exam (3h) EBA11805 - Mathematics for Data Science

12 Des. 2023

The exam set has 2 pages. All 12 problems have equal weight. You are required to give reasons for all answers. Grades: A - F which counts for 40% of the final grade in the course.
Support materials permitted: BI-approved exam calculator. Ruler.

Problem 1

Calculate the sum

$$7\,000 \cdot 1.004^{91} + 7\,000 \cdot 1.004^{90} + 7\,000 \cdot 1.004^{89} + \ldots + 7\,000 \cdot 1.004^{21} + 7\,000 \cdot 1.004^{20}$$

Problem 2

Calculate the expression for the derivative function $f'(x)$ if

i) $f(x) = 12xe^x$ ii) $f(x) = \dfrac{36-4x}{x-7}$ iii) $f(x) = \ln(x^{50})$

Problem 3

We have the function $f(x) = \frac{36-4x}{x-7}$.

i) Determine the asymptotes for $f(x)$.
ii) Draw a sketch of the graph of $f(x)$ with asymptotes.

Problem 4

i) Solve the inequality $\ln(x+5) \geqslant 3$.
ii) Determine which of the numbers $1.24^{100\,000}$ and $1.02^{1\,200\,000}$ is the larger.

Problem 5

Here is a cash flow:

Year	0	5	6
Payment	−20	10	25

i) Write up the equation for the internal rate of return. (Note: you are not supposed to solve the equation!)
ii) Determine whether the internal rate of return is larger or smaller than 10% (Note: calculation by the finance buttons on the calculator is not a valid argument!).

Problem 6

Draw a sketch of the graphs of the functions $f(x)$ and $g(x)$ with the given data. (Note: you are not supposed find any function expressions!)

i) $f(3) = 10$, $f(9) = 15$, $f(15) = 20$, $f'(6) = -1$, $f'(12) = 1$.
ii) $g'(x) < 0$ for all x between 20 and 100, $g''(x) > 0$ for all x between 20 and 60, $g''(x) < 0$ for all x between 60 and 100.

Problem 7

In figure 1 you see the graph of the derivative function $f'(x)$.

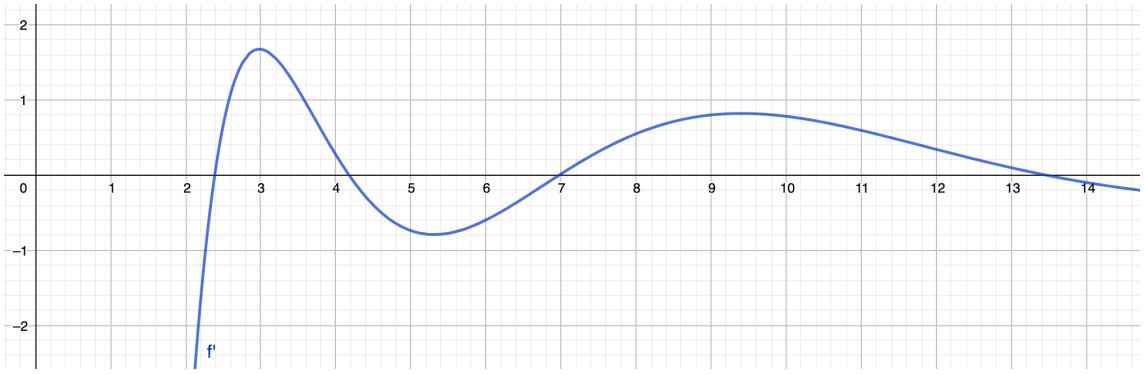

Figure 1: Graph of $f'(x)$

Determine whether the statement is true or false. Give a short explanation.

i) $f(9) > f(11)$.
ii) $f(x)$ has three stationary points in the interval $[2, 8]$.
iii) $f(x)$ has three inflection points in the interval $[2, 11]$.

Problem 8

We have the cost function $C(x) = 3x + 100e^{rx}$ with $x \geqslant 0$ where r is an undetermined positive number (a parameter).

i) Determine the marginal cost function.
ii) Determine the cost optimum and the minimal average unit cost.

Problem 9

Let p be the price of a commodity and suppose $D(p) = 7(p-60)^2$ with $0 < p < 60$ is the demand function. Suppose $\varepsilon(p)$ is the momentary price elasticity of the demand function.

i) Calculate $\varepsilon(p)$.
ii) Determine whether the revenue is increasing or decreasing if the price increases a little from $p = 24$.

Problem 10

i) Find the Taylor polynomial $P_2(x)$ of degree 2 about $x = 9$ for the function $f(x) = \sqrt{x}$.
ii) Use $P_2(x)$ to determine an approximate value for $\sqrt{10}$.

Problem 11

We have a curve C given by the equation

$$x^3 - 4xy + y^2 = 0$$

i) Use implicit differentiation to express y' in terms of y and x.
ii) Determine all solutions for y given that $x = 3$ and determine the slopes of the tangents to C in those points.

Problem 12

We have the function $f(x) = \frac{100\ln(x)}{5\ln(x)+10}$ with domain of definition $D_f = \langle e^{-2}, \rightarrow \rangle$.

i) Determine the asymptotes of $f(x)$.
ii) Determine the inverse function $g(x)$ with domain of definition D_g and range R_g.