Course paper	EBA 1180 Mathematics for Data Science
Deadline	March 20th 2023 at 12.00

The problem set consists of two pages. All sub-questions have equal weight. A score of at least 60% is required to pass. **You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.** Your answers should be provided as a single file in PDF format, for instance by writing by hand and then scanning your answers.

Problem 1.
Compute:

a) $\displaystyle\int_0^7 x^2\sqrt{x}\,\mathrm{d}x$ b) $\displaystyle\int_1^2 \ln(\sqrt{x})\,\mathrm{d}x$ c) $\displaystyle\int_1^2 \frac{6}{x^2-9}\,\mathrm{d}x$ d) $\displaystyle\int_0^1 \frac{\sqrt{x}}{\sqrt{x}+1}\,\mathrm{d}x$

e) $\displaystyle\int_{-1}^0 x\sqrt{-x}\,\mathrm{d}x$ f) $\displaystyle\int_{-1}^1 x\sqrt{|x|}\,\mathrm{d}x$ g) $\displaystyle\int_1^{e^2} \frac{\sqrt{\ln x}}{x}\,\mathrm{d}x$

Problem 2.
The parabola P intersects the x-axis in $x = 2 \pm \sqrt{5}$ and has a vertex with $y = 5$. The hyperbola H has asymptotes $x = 0$ and $y = 0$, and intersects P in $x = 1$.

a) Find the equation of the parabola P and of the hyberbola H, and draw P and H in the same coordinate system.
b) Let R be the area in the first quadrant bounded by P and H. Find the area of R.

Problem 3.
Let the continuous function $f(t) = 100 \cdot e^{\sqrt{t}}$ be a model for a continuous cash flow (in million NOK per year) after t years. Find the total cash flow for the first 25 years. Find an expression for the present value of this cash flow when we use continuous discounting with discount rate r.

Problem 4.
Use Gaussian elimination to solve the linear system $A\mathbf{x} = \mathbf{b}$. Show the elementary row operations, mark the pivot positions of the echelon form and specify the number of solutions.

a) $(A|\mathbf{b}) = \left(\begin{array}{cccc|c} 2 & 1 & 2 & -3 & 4 \\ 3 & -1 & 8 & 2 & 7 \\ 5 & 5 & 0 & -17 & 12 \end{array}\right)$ b) $(A|\mathbf{b}) = \left(\begin{array}{ccc|c} 1 & 3 & -3 & 2 \\ 3 & 1 & 2 & 4 \\ 2 & -1 & 4 & 3 \\ 4 & 5 & 1 & 13 \end{array}\right)$

Problem 5.
The linear system $A\mathbf{x} = \mathbf{b}$ is given by

$$A = \begin{pmatrix} a & 2 & 3 \\ 2 & a & 3 \\ 2 & 3 & a \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

where a is a parameter.

a) Compute $|A|$.
b) Find A^{-1} when $a = 0$, and use this to solve the linear system in this case.
c) Determine all values of a such that $A\mathbf{x} = \mathbf{b}$ has a unique solution.
d) Find all solutions of the linear system in the cases where there are infinitely many solutions.

Problem 6.
Consider the following 3-vectors:

$$\mathbf{v}_1 = \begin{pmatrix} 5 \\ 4 \\ 7 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 5 \\ 8 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 3 \\ 8 \\ 13 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

a) Write the vector $\mathbf{v}_3$ as a linear combination of the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ if possible.
b) Determine all values of a,b,c such that $\mathbf{w}$ is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and $\mathbf{v}_4$.
c) Determine all vectors $\mathbf{w}$ such that $\mathbf{w} \perp \mathbf{v}_2$.

Problem 7.
Solve the matrix equation $XA = AX$ for X when

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Problem 8.
Consider the function $f(x,y) = x^2 - 4xy + 5y^2 - 4x + 4y + 1$.

a) Find the stationary points for f, if there are any, and classify these.
b) Determine whether f has maximum- or minimum values.

Problem 9.
Consider the function $f(x,y) = x^2y^3 + y^2 - 2y + 1$.

a) Find the stationary points for f, if there are any, and classify these.
b) Determine whether f has maximum- or minimum values.