

Exercise session problems

Problem 1.

Compute the following inner products when

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad \vec{v}_4 = \begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$$

- a) $\vec{v}_1 \cdot \vec{v}_2$ b) $\vec{v}_1 \cdot \vec{v}_3$ c) $\vec{v}_2 \cdot \vec{v}_2$ d) $(\vec{v}_1 - \vec{v}_2) \cdot \vec{v}_4$
 e) $\vec{v}_1 \cdot (\vec{v}_2 + \vec{v}_3)$ f) $\vec{v}_1 \cdot (\vec{v}_2 - \vec{v}_3)$ g) $(\vec{v}_4 - \vec{v}_1) \cdot (\vec{v}_2 + \vec{v}_3)$ h) $(\vec{v}_1 - \vec{v}_2) \cdot (\vec{v}_1 - \vec{v}_2)$

Problem 2.

Find all the vectors that are orthogonal to the vector \mathbf{v} :

a) $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ b) $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ c) $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ d) $\mathbf{v} = \begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$

Problem 3.

Determine $\|\vec{v} - \vec{w}\|$ when the vectors \vec{v}, \vec{w} are orthogonal and have length $\|\vec{v}\| = 3$ and $\|\vec{w}\| = 4$.

Problem 4.

Find the natural domain D_f and range V_f of the function f :

a) $f(x,y) = 2x + 3y$ b) $f(x,y) = \sqrt{x+3y}$ c) $f(x,y) = (2x - y)^{-3/2}$ d) $f(x,y) = 17x^{1.2}y^{3.4}$

Problem 5.

Consider the level curve $f(x,y) = c$ of a function $f(x,y)$. Draw the level curves for the given values of c in the same coordinate system and determine what kind of curve we get when we let c be an arbitrary value:

a) $f(x,y) = 12x - 3y$ og $c = -3, 0, 3$ b) $f(x,y) = xy$ og $c = -1, 0, 1$
 c) $f(x,y) = x^2 + 2x + y^2 - 4y$ og $c = -9, -5, -1$ d) $f(x,y) = x^2 - 2x + 4y^2$ og $c = -2, -1, 0, 1$

Problem 6.

Use the level curves $f(x,y) = c$ from Problem 5 to determine whether the following functions have maximum- or minimum values:

a) $f(x,y) = 12x - 3y$ b) $f(x,y) = xy$
 c) $f(x,y) = x^2 + 2x + y^2 - 4y$ d) $f(x,y) = x^2 - 2x + 4y^2$

Problem 7.

Describe the graph of $f(x,y) = 3x - 4y + 1$ geometrically. What is meant by a geometrical description is for example: *The graph of $f(x) = 3 - 2x$ is a straight line with slope -2 which intersects the y -axis in $y = 3$, i.e., a precise geometrical description without using equations etc.*

Problem 8.

Find the partial derivatives f'_x and f'_y when

a) $f(x,y) = 2x + 3y$

b) $f(x,y) = x^2 + y^2$

c) $f(x,y) = 4x^2 - 6xy + 9y^2$

d) $f(x,y) = x^2 - 2x + 4y^2$

e) $f(x,y) = x^3 - 3xy + y^3$

f) $f(x,y) = y^2 - x^3 + 3x$

g) $f(x,y) = x^2y^2 - x^2 - y^2 + 3$

h) $f(x,y) = \sqrt{x^2 + y^2}$

Problem 9.

Find the Hessian matrix $H(f)$, and compute $H(f)(1,1)$:

a) $f(x,y) = 2x + 3y$

b) $f(x,y) = x^2 + y^2$

c) $f(x,y) = 4x^2 - 6xy + 9y^2$

d) $f(x,y) = x^2 - 2x + 4y^2$

e) $f(x,y) = x^3 - 3xy + y^3$

f) $f(x,y) = y^2 - x^3 + 3x$

g) $f(x,y) = x^2y^2 - x^2 - y^2 + 3$

h) $f(x,y) = \sqrt{x^2 + y^2}$

Problem 10.

Find the stationary points of f , and classify them:

a) $f(x,y) = 2x + 3y$

b) $f(x,y) = x^2 + y^2$

c) $f(x,y) = 4x^2 - 6xy + 9y^2$

d) $f(x,y) = x^2 - 2x + 4y^2$

e) $f(x,y) = x^3 - 3xy + y^3$

f) $f(x,y) = y^2 - x^3 + 3x$

g) $f(x,y) = x^2y^2 - x^2 - y^2 + 3$

h) $f(x,y) = \sqrt{x^2 + y^2}$

Problem 11.

Find all stationary points and classify them:

a) $f(x,y) = xy(x^2 - y^2)$

b) $f(x,y) = x^2y + xy^3 + xy^2$

c) $f(x,y) = \sqrt{36 - 9x^2 - 4y^2}$

Oppgaver fra læreboken

Læreboken [E]: Eriksen, *Matematikk for økonomi og finans*

Oppgaveboken [O]: Eriksen, *Matematikk for økonomi og finans - Oppgaver og Løsningsforslag*

Oppgaver: [E] 7.1.1 - 7.1.4, 7.2.1 - 7.2.2, 7.3.1 - 7.3.5, 7.4.1 - 7.4.2

Fullstendig løsning: Se [O] Kap 7.1 - 7.4

Answers to the exercise session problems

Problem 1.

- a) 0 b) 4 c) 2 d) 13
e) 4 f) -4 g) 18 h) 5

Problem 2.

All linear combinations of the following vectors:

- a) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ b) $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ c) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ d) $\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} -7 \\ 4 \\ 0 \end{pmatrix}$

Problem 3.

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Problem 4.

- a) $D_f = \mathbb{R}^2, V_f = \mathbb{R}$ b) $D_f = \{(x,y) \in \mathbb{R}^2 : x + 3y \geq 0\}, V_f = [0, \infty)$
c) $D_f = \{(x,y) \in \mathbb{R}^2 : 2x - y > 0\}, V_f = (0, \infty)$ d) $D_f = \mathbb{R}^2, V_f = \mathbb{R}$

Problem 5.

- a) Straight line with slope 4 which intersects the y -axis in $y = -c/3$
b) Hyperbola $y = c/x$ if $c \neq 0$, and the two axis if $c = 0$
c) Circle with radius $\sqrt{c+5}$ and center $(-1,2)$ if $c > -5$, one point $(-1,2)$ if $c = -5$, no points otherwise.
d) Ellipse with center in $(1,0)$ with half axis $a = \sqrt{c+1}$ and $b = \sqrt{c+1}/2$ when $c > -1$, one point $(1,0)$ if $c = -1$, and no points otherwise.

Problem 6.

- a) Neither maximum nor minimum.
b) Neither maximum nor minimum.
c) No maximum, but the minimum value is $f_{min} = -5$.
d) No maximum, but the minimum value is $f_{min} = -1$

Problem 7.

The graph is the plane which intersects the z -axis in $z = 1$ and has the normal vector $(3, -4, -1)$.

Problem 8.

- a) $f'_x = 2, f'_y = 3$ b) $f'_x = 2x, f'_y = 2y$ c) $f'_x = 8x - 6y, f'_y = -6x + 18y$
d) $f'_x = 2x - 2, f'_y = 8y$ e) $f'_x = 3x^2 - 3y, f'_y = -3x + 3y^2$ f) $f'_x = -3x^2 + 3, f'_y = 2y$
g) $f'_x = 2x(y^2 - 1), f'_y = 2y(x^2 - 1)$ h) $f'_x = \frac{x}{\sqrt{x^2 + y^2}}, f'_y = \frac{y}{\sqrt{x^2 + y^2}}$

Problem 9.

a) $H(f) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, H(f)(1,1) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

b) $H(f) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, H(f)(1,1) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

c) $H(f) = \begin{pmatrix} 8 & -6 \\ -6 & 18 \end{pmatrix}, H(f)(1,1) = \begin{pmatrix} 8 & -6 \\ -6 & 18 \end{pmatrix}$

d) $H(f) = \begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}, H(f)(1,1) = \begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$

e) $H(f) = \begin{pmatrix} 6x & -3 \\ -3 & 6y \end{pmatrix}, H(f)(1,1) = \begin{pmatrix} 6 & -3 \\ -3 & 6 \end{pmatrix}$

f) $H(f) = \begin{pmatrix} -6x & 0 \\ 0 & 2 \end{pmatrix}, H(f)(1,1) = \begin{pmatrix} -6 & 0 \\ 0 & 2 \end{pmatrix}$

g) $H(f) = \begin{pmatrix} 2(y^2 - 1) & 4xy \\ 4xy & 2(x^2 - 1) \end{pmatrix}, H(f)(1,1) = \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}$

h) $H(f) = (x^2 + y^2)^{-3/2} \cdot \begin{pmatrix} y^2 & -xy \\ -xy & x^2 \end{pmatrix}, H(f)(1,1) = \begin{pmatrix} \sqrt{2}/4 & -\sqrt{2}/4 \\ -\sqrt{2}/4 & \sqrt{2}/4 \end{pmatrix}$

Problem 10.

- a) None b) (0,0) is a local min. c) (0,0) is a local min min. d) (1,0) is a local min min.
 e) (0,0) is a saddle point and (1,1) is a local min. f) (1,0) is a saddle point and (-1,0) is a local min.
 g) (0,0) is a local max and $(\pm 1, \pm 1)$ is a saddle point. h) none; (0,0) is a critical point.

Problem 11.

- a) (0,0) is a saddle point. b) (0,0), (0, -1) is a saddle point, $(3/25, -3/5)$ is a local max.
 c) (0,0) is a local (and global) max.