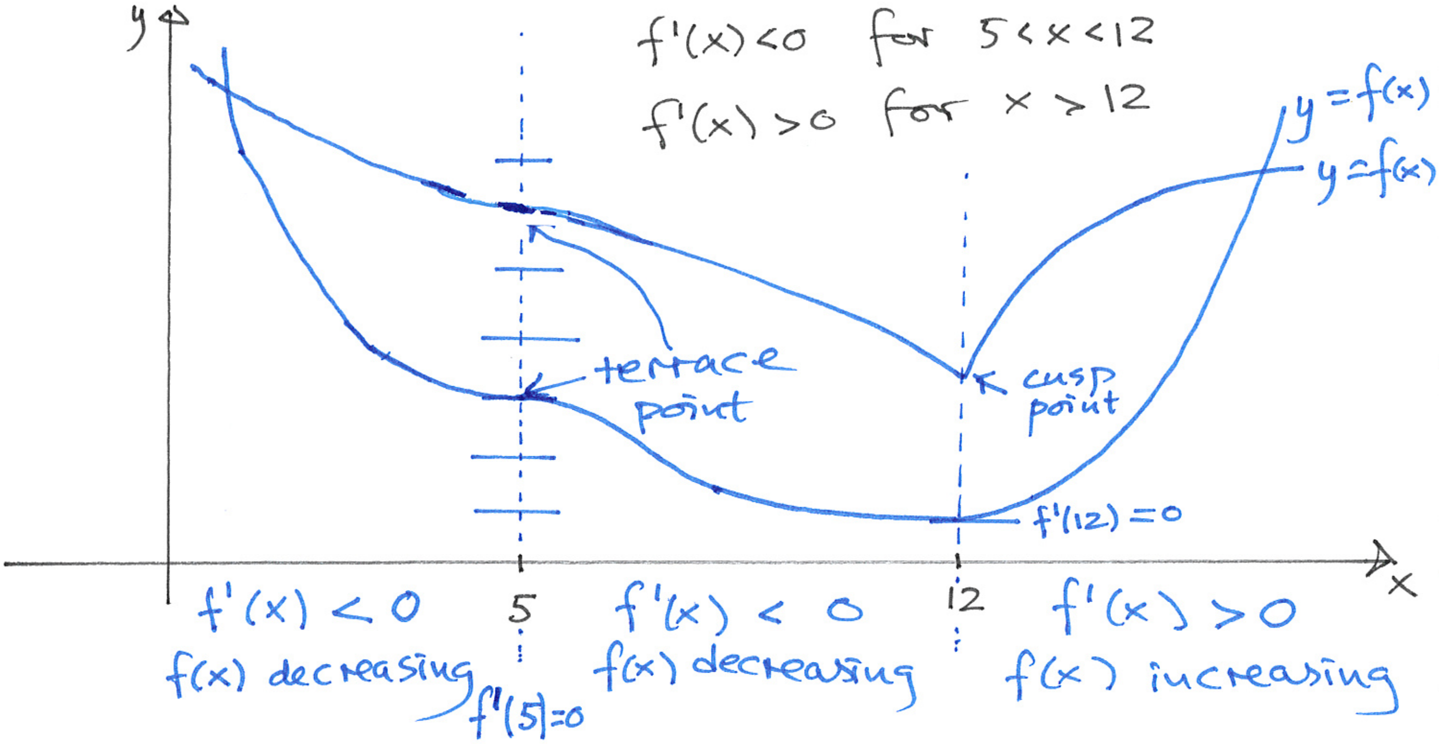
Plan
1. Repetition with problems from last week:
   - Probl. 1c: draw two graphs
   - Probl. 2 b, d, h, i, k
     interpretations of the graph of $f'(x)$.
   - Probl. 3c: which graph is $f(x)$ / $f'(x)$?
   - Probl. 4g: increasing/decreasing from $f'(x)$.
2. Implicit differentiation

1. Repetition **Probl 1c**

$f'(x) < 0$ for $x < 5$, $f'(5) = 0$

$f'(x) < 0$ for $5 < x < 12$

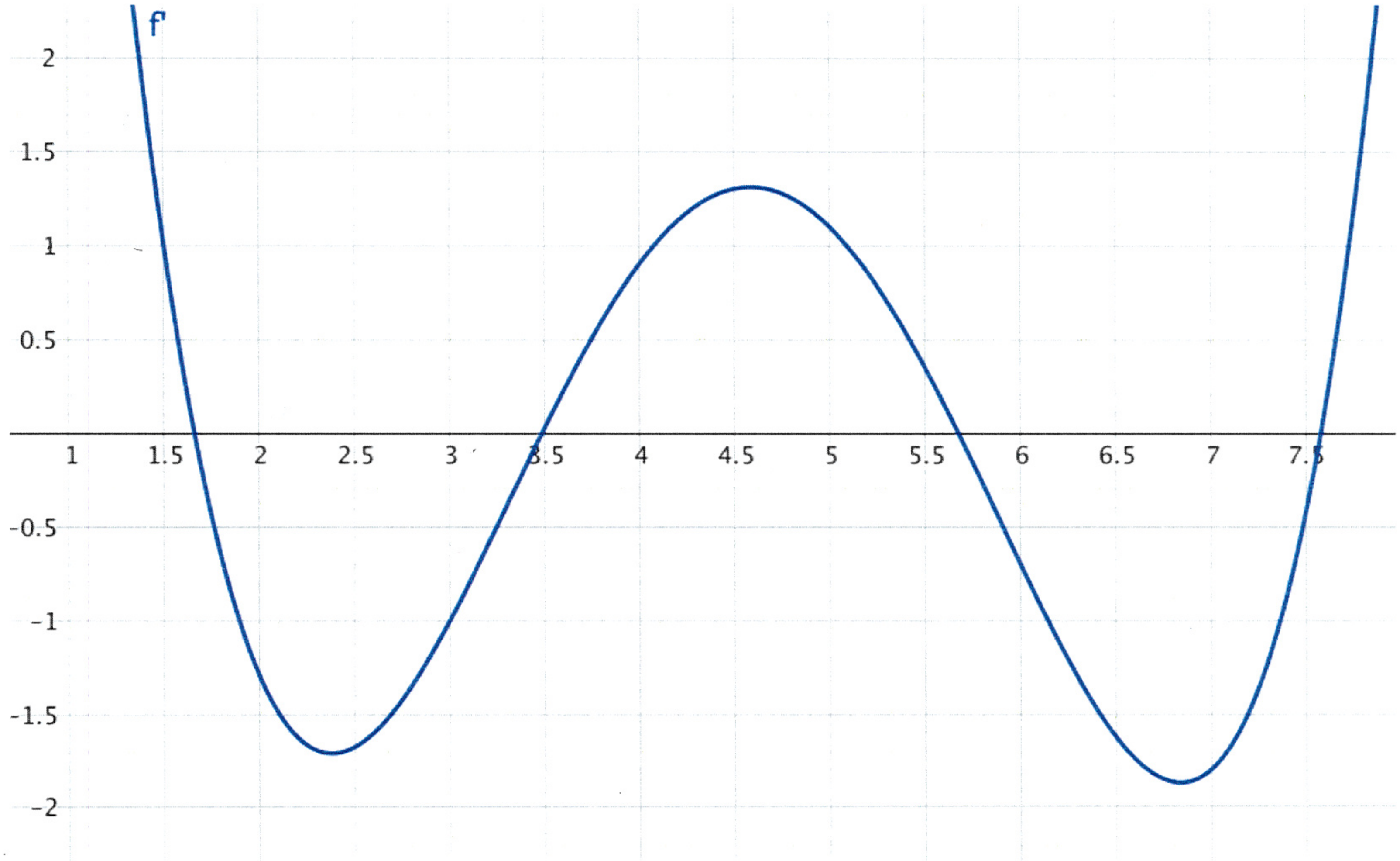
$f'(x) > 0$ for $x > 12$



Probl 2 b) $f(2) < f(3)$ FALSE.

We see (from the graph of $f'(x)$) that $f'(x) < 0$ for $x \in [2, 3]$. Hence $f(x)$ is strictly decreasing in $[2, 3]$ and in particular $f(2) > f(3)$.

Oppgave 2 I figur 1 ser du grafen til $f'(x)$.



Figur 1: Grafen til $f'(x)$

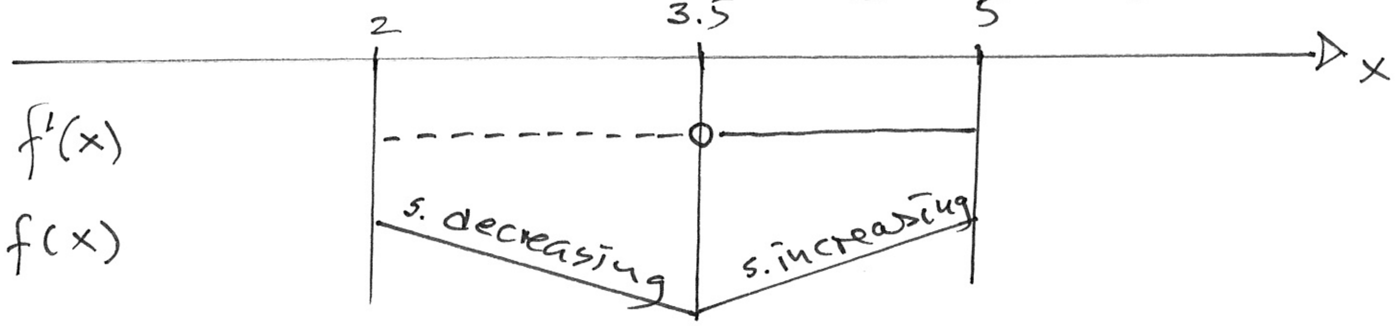
2d) $f(x)$ has a (local) minimum at $x = 3.5$.

TRUE because:

We have $f'(x) < 0$ for $x \in [2, 3.5\rangle$

and $f'(x) > 0$ for $x \in \langle 3.5, 5]$

and $f'(3.5) = 0$. Sign diagram:



Conclusion: $x = 3.5$ is a loc. min. point for $f(x)$.

2h) $f(x)$ increases faster around $x = 1.5$ than around $x = 5.5$. TRUE.

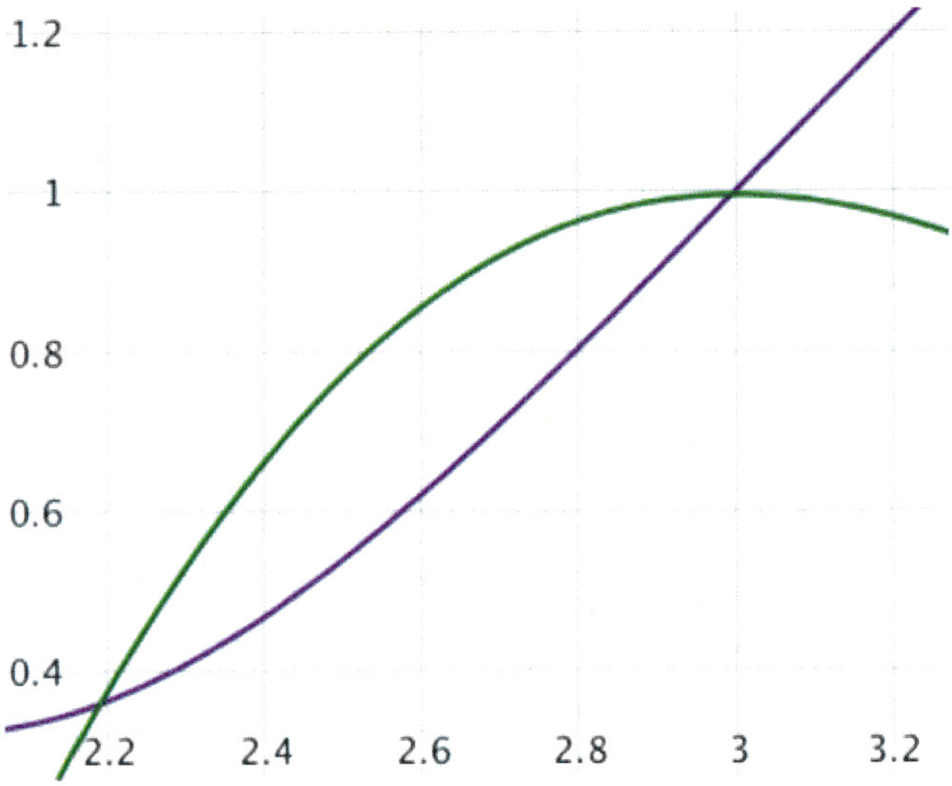
The slope of the tangent of $f(x)$ at $x = 1.5$ is approx. 1 (since $f'(1.5) \approx 1$)

The slope of the tangent of $f(x)$ at $x = 5.5$ is approx 0.35 (since $f'(5.5) \approx 0.35$)

2i) The derivative of $f'(x)$ is positive for $x = 7.6$.

TRUE because the slope of the tangent of $f'(x)$ is (very) positive for $x = 7.6$

(maybe $f''(7.6) \approx 6$)


1.2
1
0.8
0.6
0.4
2.2
2.4
2.6
2.8
3
3.2

2k) We cannot use the graph of $f'(x)$ to determine if $f(4.5)$ is positive.

TRUE: If we add or subtract 1 mill to $f(x)$, $f'(x)$ is not changed.

Probl. 3c Which graph is $f(x) / f'(x)$?

I guess $f(x)$ is the violet one. But much easier to determine what is wrong!

Therefore I assume that $f(x)$ is the green. Then $f'(x)$ is the violet. But the slope of the tangent of green is negative for $x > 3$ while the values of the violet ~~is~~ are bigger than 1. So the assumption is wrong. The only possibility is that $f(x)$ is the violet and $f'(x)$ is the green.

Probl 4g $f'(x) = e^{2x} - 4e^{x} + 3$

Find the stationary points for $f(x)$ and determine where $f(x)$ is increasing/decreasing.

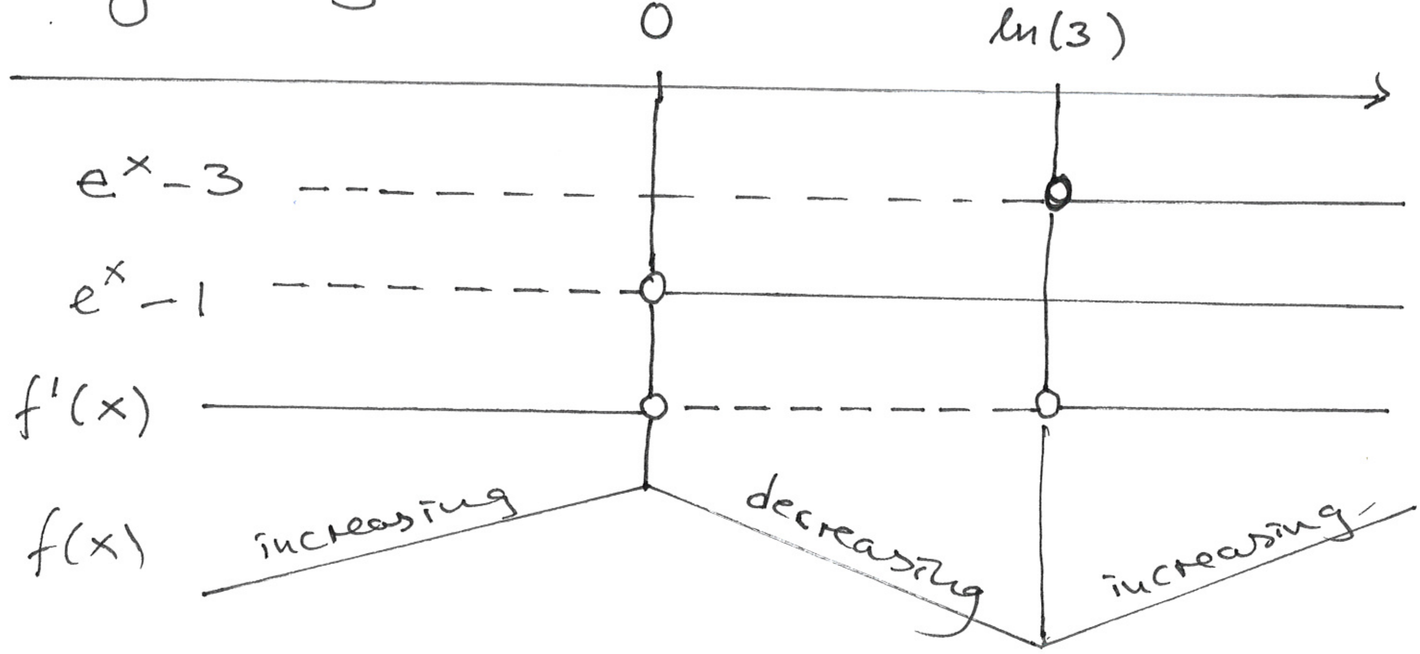
Want to use a sign diagram for $f'(x)$. But we have factorise $f'(x)$ first. Put $u = e^x$ and get $u^2 = (e^x)^2 = e^{2x}$

Start: 11.00

So $f'(x) = u^2 - 4u + 3 = (u-3)(u-1)$

$f'(x) = (e^x - 3)(e^x - 1)$.

Sign diag:



So $f(x)$ is strictly increasing for x in $\langle \leftarrow, 0]$

— " — decreasing — " — $[0, \ln(3)]$

— " — increasing — " — $[\ln(3), \rightarrow\rangle$

Stationary points for $f(x)$ are the zeros of $f'(x)$: $\underline{\underline{x = 0}}$, $\underline{\underline{x = \ln(3)}}$

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2. Implicit differentiation

Ex $f(x) = \frac{1}{x} = x^{-1}$, $f'(x) = (-1) \cdot x^{-1-1} = -x^{-2} = -\frac{1}{x^2}$

– usual differentiation.

Instead put $y = f(x)$, so $y = \frac{1}{x}$ $1 \cdot x$

and get $\boxed{xy = 1}$ – eq with both x and y.

Differentiate each side of the eq. with respect to x and think about y as a function of x, so $y = y(x)$.

$$(x \cdot y)'_x = (1)'_x$$

the product rule on the LHS gives

$$(x)'_x \cdot y + x \cdot (y)'_x = 0$$

$$1 \cdot y + x \cdot y'_x = 0$$

We can solve this for y'_x :

$$x \cdot y' = -y \quad | : x$$

$$\boxed{y' = -\frac{y}{x}}$$

(Note that $y = \frac{1}{x}$ so $y' = -\frac{\frac{1}{x}}{x} = -\frac{1}{x^2}$)

This is called implicit differentiation.

<u>One application</u> Can use this to find slopes of tangents to the curve defined by the original equation ($xy = 1$)

E.g. if $x = 2$ then $xy = 1$ gives $2y = 1 \quad : 2$

so $y = \frac{1}{2}$

Then $y'\Big|_{\substack{x=2 \\ y=\frac{1}{2}}} \overset{\text{red box}}{=} -\frac{\frac{1}{2}}{2} = -\frac{1}{4}$

Can apply this to find the ~~equation~~ expression $h(x)$ of the tangent of the curve $xy = 1$ at the point $(2, \frac{1}{2})$ by the point-slope formula:

$$h(x) - \frac{1}{2} = -\frac{1}{4}(x - 2)$$

the slope

so $h(x) = -\frac{1}{4} \cdot x + 1$