

**EBA1180 Mathematics for Data Science
autumn 2024
Exercises**

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

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Lecture 8 and 9

Sec. 4.7, 3.4-5, 2.6:

Polynomial division. Factorisation. Rational and radical equations. Inequalities.

Here are recommended exercises from the textbook [SHSC].

Section 4.7 exercise 1-6

Section 3.4 exercise 1, 2a, b, d

Section 3.5 exercise 2-4

Section 2.6 exercise 1-3, 5

**Problems for the exercise session
Wednesday 18 Sept. 12-16+ in CU1-067**

Problem 1 Perform the polynomial division with remainder.

- a) $(x^2 + 4x - 21) : (x - 3)$ b) $(x^2 + 4x - 21) : (x - 4)$ c) $(x^3 + x^2 - 23x + 42) : (x + 6)$
d) $(x^3 + x^2 - 23x + 42) : (x + 1)$ e) $(x^4 + x^2 + 1) : (x^2 - x + 1)$ f) $(x^2 + 3x - 7) : (x - a)$

Problem 2 Factorise the polynomial into factors of the least possible degree (by guessing on a zero and performing polynomial division).

- a) $x^2 + 4x - 221$ b) $x^3 + 6x^2 - x - 30$ c) $x^3 - 3x^2 + 5x - 15$
d) $x^4 + 10x^3 + 35x^2 + 50x + 24$

Problem 3 Solve the equations.

- a) $\frac{5x + 1}{x^2 + x + 1} = -2$ b) $\frac{x - 1}{x^2 + x + 1} = 1$ c) $\frac{1}{1 + \frac{1}{x}} = \frac{1}{x}$ d) $\frac{(x - 1)(x - 3)}{(x - 2)(x - 4)} = 2$

Problem 4 Determine the values of a such that the equation has solutions.

- a) $x^2 + 2ax + 9 = 0$ b) $\frac{1}{x + a} = \frac{2}{2x + 3}$ c) $\frac{(x - 1)(x - 3)}{(x - 2)(x - 4)} = a$

Problem 5 Solve the equations.

a) $\sqrt{2x+3} = x+2$

b) $\sqrt{4x+1} = x-1$

c) $\sqrt{x+2} + \sqrt{x-3} = 5$

d) $\sqrt{2x+1} - \sqrt{x+4} = 1$

e) $\frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} = 2$

f) $\frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} = -1$

Problem 6 Determine the values of a such that the equation $\frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} = a$ has solutions.
(Note problem 5e and 5f.)

Problem 7 Solve the inequalities.

a) $2x+3 \leq 5x+2$

b) $-4x+1 \geq x-1$

c) $x+2 < 3+5x$

d) $(x-5)(x+4) < 0$

e) $(2x+5)(7-x) \geq 0$

f) $\frac{x-5}{x+4} \geq 0$

g) $\frac{(x-2)(x+3)}{(x-5)(x+4)} < 0$

h) $\frac{-5}{(6-x)(-12-3x)} \geq 0$

i) $(x-5)(x+4) < 10$

j) $(2x+5)(7-x) \geq 35$

k) $\frac{(x-2)(x+3)}{(x-5)(x+4)} < 1$

l) $\frac{-5}{(6-x)(-12-3x)} \geq \frac{5}{72}$

Problem 8 Determine a such that the inequality has solutions.

a) $x^2 + 6x \leq a$

b) $(x+a)^2 < a$

Answers

Problem 1

- a) $x + 7$ b) $x + 8 + \frac{11}{x-4}$ c) $x^2 - 5x + 7$
 d) $x^2 - 23 + \frac{65}{x+1}$ e) $x^2 + x + 1$ f) $x + a + 3 + \frac{a^2 + 3a - 7}{x-a}$

Problem 2

- a) $(x-13)(x+17)$ b) $(x-2)(x+3)(x+5)$ c) $(x-3)(x^2+5)$
 d) $(x+1)(x+2)(x+3)(x+4)$

Problem 3

- a) $x = -3, x = -\frac{1}{2}$ b) no solutions c) $x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$ d) $x = 4 \pm \sqrt{3}$

Problem 4

- a) $a \leq -3$ or $a \geq 3$ b) $a = \frac{3}{2}$ c) All values of a give solutions

Problem 5

- a) $x = -1$ b) $x = 6$ c) $x = 7$
 d) $x = 12$ e) $x = 2$ f) no solutions

Problem 6

$a \leq -2$ or $a > 0$ (compare with 5e and 5f)

Problem 7

- a) $x \geq \frac{1}{3}$, alternative way of writing: $x \in [\frac{1}{3}, \infty)$
 b) $x \leq \frac{2}{5}$, alternative: $x \in (-\infty, \frac{2}{5}]$
 c) $x > -\frac{1}{4}$, alternative: $x \in (-\frac{1}{4}, \infty)$
 d) $-4 < x < 5$, alternative: $x \in (-4, 5)$
 e) $-\frac{5}{2} \leq x \leq 7$, alternative: $x \in [-\frac{5}{2}, 7]$
 f) $x < -4$ or $x \geq 5$, alternative: $x \in (-\infty, -4) \cup [5, \infty)$
 g) $-4 < x < -3$ or $2 < x < 5$, alternative: $x \in (-4, -3) \cup (2, 5)$
 h) $-4 < x < 6$, alternative: $x \in (-4, 6)$
 i) $-5 < x < 6$, alternative: $x \in (-5, 6)$
 j) $0 \leq x \leq \frac{9}{2}$, alternative: $x \in [0, \frac{9}{2}]$
 k) $x < -7$ eller $-4 < x < 5$, alternative: $x \in (-\infty, -7) \cup (-4, 5)$
 l) $-4 < x \leq 0$ or $2 \leq x < 6$, alternative: $x \in (-4, 0] \cup [2, 6)$

Problem 8

- a) $a \geq -9$ b) $0 < a$