Exercise session problems

Problem 1.
Consider the Lagrange problem: $\max / \min \; f(x,y) = xy \quad \text{when} \quad x^2 + y^2 = 4$

a) Solve the Lagrange conditions and find the corresponding candidate points.
b) Are there ponits with a degenerate constraint?
c) Solve the optimization problem.

Problem 2.
Assume that the Lagrange problem $\max f(x,y)$ when $g(x,y) = 4$ has a maximum value $f(1,3) = 12$ in the ordinary candidate point $(x,y;\lambda) = (1,3;2)$. What is the interpretation of $\lambda = 2$? Use this to estimate the maximum value of the Lagrange problem $\max f(x,y)$ when $g(x,y) = 3$.

Problem 3.
What does it mean for a constraint in a Lagrange problem to be degenerate? Can you give examples of a constraint $g(x,y) = a$ which has an admissible point with a degenerate constraint? Can you find a function $f(x,y)$ such that the optimization problem $\max f(x,y)$ when $g(x,y) = a$ has the point with a degenerate constraint as its maximum point?

Problem 4.
Consider the Lagrange problem: $\min f(x,y) = xy \quad \text{when} \quad x^2 + 4y^2 = 4$.

a) Sketch the curve given by $x^2 + 4y^2 = 4$, and determine whether this set is bounded.
b) Write down the Lagrange constraints and find all $(x,y;\lambda)$ which satisfy these constraints.
c) Solve the Lagrange problem.
d) Give an interpretation of the Lagrange multiplier of a Lagrange problem, and use this interpretation to estimate the minimum value of the new Lagrange problem: $\min f(x,y) = xy \quad \text{when} \quad x^2 + 4y^2 = 5$

Problem 5.
Consider the function $f(x,y) = x^2y^2 + xy + x - y$.

a) Show that the level curve $f(x,y) = 2$ intersects the line $y = x$ in two points (a,a) and (b,b).
b) Find the tangent of the level curve $f(x,y) = 2$ in the points (a,a) and (b,b).
c) Find any stationary points for f, and classify these as local maxima, local minima or saddle points.

Answers to exercise session problems

Problem 1.

a) $(\pm\sqrt{2}, \pm\sqrt{2}; 1/2)$, $(\pm\sqrt{2}, \mp\sqrt{2}; -1/2)$

b) No

c) $f_{\max} = 2$, $f_{\min} = -2$

Problem 2.

$f_{\max} \approx 12 + (-1) \cdot 2 = 10$

Problem 4.

a) Bounded (ellipsis)

b) $(\sqrt{2}, \sqrt{2}/2; 1/4), (-\sqrt{2}, -\sqrt{2}/2; 1/4), (\sqrt{2}, -\sqrt{2}/2; -1/4), (-\sqrt{2}, \sqrt{2}/2; -1/4)$

c) $f_{\min} = -1$

d) $f_{\min} \approx -1.25$ when $x^2 + 4y^2 = 5$

Problem 5.

a) (1,1),$(-1, -1)$

b) $y = 2x - 3$, $y = -x/2 - 3/2$

c) $(-1,1)$, saddle point