Exercise session problems

Problem 1.
Solve the optimization problem. Illustrate the set of admissible points D, along with suitable level curves for f in the same coordinate system:

a) $\max / \min f(x,y) = x^2 + y^2$ when $x + y = 2$
b) $\max / \min f(x,y) = 4x^2 + 9y^2$ when $2x + 3y = 6$
c) $\max / \min f(x,y) = y$ when $x^2 - y^2 = 1$

Problem 2.
Solve the optimization problem: $\max / \min \; f(x,y) = x^3 + 3xy + y^3 \quad \text{when} \quad xy = 1$

Problem 3.
Consider the curve C given by the equation $y(x^2 + y^2) = 2(x^2 - y^2)$.

a) Find all points on the curve C where $y = -1$.
b) Find the tangent of C in each point where $y = -1$.
c) Solve the optimization problem: $\max / \min \; f(x,y) = y \quad \text{when} \quad y(x^2 + y^2) = 2(x^2 - y^2)$

Problem 4.
Consider the function defined by $f(x,y) = 1 + x^2 + y^2 + x^2y^2$.

a) Find all stationary points for f.
b) Compute the Hessian of f, and use this to classify the stationary points.
c) Determine whether f has global maximum- or minimum values.
d) Solve the Lagrange problem: $\max f(x,y) = x^2 + y^2 + x^2y^2$ when $x^2 + 2y^2 = 5$

Problem 5.
Consider the Lagrange problem $\max / \min f(x,y) = x^2 - xy + y^2$ when $x + y = 2$.

a) Use the Lagrange multiplier method to find candidates $(x,y;\lambda)$ for the maximum and minimum.
b) Write the function $f(x,y)$ by using that $(x + y)^2 = 2^2 = 4$ in all admissible points (i.e., all points that satisfy the constraint). Use the Lagrange multiplier method to find candidates $(x,y;\lambda)$ for the maximum and minimum in this new Lagrange problem.
c) Solve the constraint for one of the variables, and use this to simplify the expression for $f(x,y)$ to a function in one variable. Solve the optimization problem you have now.
d) Compare the previous answers and discuss the connection between the three methods. Then solve the optimization problem.

Answers to exercise session problems

Problem 1.

a) $f_{\min} = 2$, no maximum value.

b) $f_{\min} = 18$, no maximum value.

c) No maximum nor minimum value.

Problem 2.
Neither maximum nor minimum exist.

Problem 3.

a) $(\pm\sqrt{1/3}, -1)$

b) $y = 2 \mp 3\sqrt{3}x$

c) $f_{\min} = -2$, no maximum value

Problem 4.

a) (0,0)

b) local minimum points

c) $f_{\min} = 1$, no maximum value

d) $f_{\max} = 7$

Problem 5.

a) $(1{,}1; 1)$

b) $(1{,}1; -3)$

c) (1,1)

d) $f_{\min} = 1$, no maximum value.