## Exercise session problems

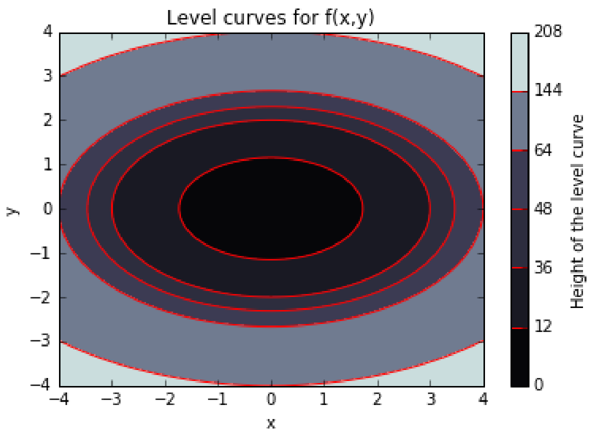
**Problem 1.**
Consider the subset $D \subseteq \mathbb{R}^2$ given by the inequality $y(x-2) \leq 3$. Make a sketch of $D = \{(x,y) : y(x-2) \leq 3\}$, and mark the inner points and the boundary points of $D$. Is $D$ compact?

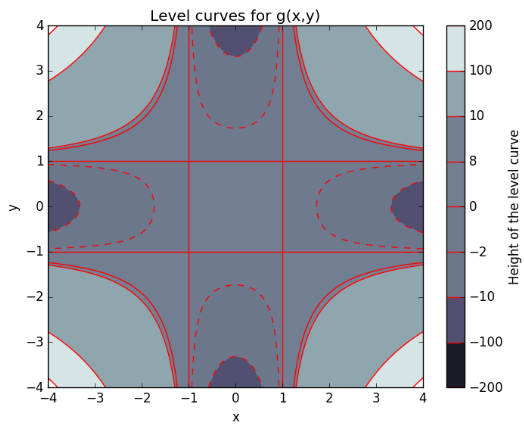
**Problem 2.**
Consider a subset of the plane $\mathbb{R}^2$ given by the following conditions. Determine whether the subset is compact. It is useful to make a sketch of the area.

a) $2x + 3y = 6$
b) $2x + 3y < 6$
c) $2x + 3y \leq 6$
d) $x^2 + y^2 = 4$
e) $x^2 + y^2 \geq 4$
f) $x^2 + y^2 \leq 4$
g) $x^2 - 2x + 4y^2 = 4$
h) $x^2 - 2x + 4y^2 \leq 4$
i) $x^2 - 2x + 4y^2 \geq 4$
j) $xy = 1$
k) $xy \leq 1$
l) $xy \geq 1$
m) $\sqrt{x^2 + y^2} = 3$
n) $\sqrt{x^2 + y^2} \leq 3$
o) $x^2y^2 - x^2 - y^2 + 1 = 0$
p) $x^2y^2 - x^2 - y^2 + 1 = 1$

**Problem 3.**
State the extreme value theorem. Give examples of a set $D$ in the plane which is closed, but not bounded, as well as a set $E$ in the plane which is bounded, but not closed. Can you find a function $f(x,y)$ which does not have a maximum nor a minimum in $D$, and a function which does not have a maximum nor a minimum in $E$?

**Problem 4.**
Level curves for the functions $f(x,y) = 4x^2 + 9y^2$ and $g(x,y) = x^2y^2 - x^2 - y^2 + 1$ in the area $-4 \leq x,y \leq 4$ are shown in the figures below.




a) Find max / min $f(x,y)$ when $-4 \leq x,y \leq 4$ by using the figure.
b) Find max / min $g(x,y)$ when $-4 \leq x,y \leq 4$ by using the figure.
c) Find max / min $f(x,y)$ when $x^2 + y^2 = 16$ by using the figure.
d) Find max / min $g(x,y)$ when $x = y$ by using the figure.

**Problem 5.**

Solve the optimization problems:

a) $\max / \min f(x,y) = x^3 - 3xy + y^3$ when $0 \leq x,y \leq 1$

b) $\max / \min f(x,y) = x^3 - 3xy + y^3$ when $0 \leq x,y \leq 2$

c) $\max / \min f(x,y) = e^{xy-x-y}$ when $0 \leq x,y \leq 2$

d) $\max / \min f(x,y) = xy(x^2 - y^2)$ when $-1 \leq x,y \leq 1$

e) $\max / \min f(x,y) = (x^2 - 1)(y^2 - 1)$ when $-1 \leq x,y \leq 1$

**Problem 6.**

Find the maximum- and minimum value for the optimization problem

$$\max / \min f(x,y) = \sqrt{xy} - x \text{ when } 0 \leq x,y \leq 1$$

## Optional: Exercises from the Norwegian textbook

| | |
|---|---|
| Textbook [E]: | Eriksen, *Matematikk for økonomi og finans* |
| Exercise book [O]: | Eriksen, *Matematikk for økonomi og finans - Oppgaver og Løsningsforslag* |

| | |
|---|---|
| Exercises: | [E] 7.6.1 - 7.6.2 |
| Solution manual: | See [O] Ch. 7.6 |

## Answers to the exercise session problems

**Problem 1.**

Boundary points are given by the equation $y(x-2) = 3$, that is points on the graph of $y = 3/(x-2)$ (a hyperbola). Inner points are given by $y(x-2) < 3$, that is points under the hyperbola when $x > 2$, and points over the hyperbola when $x < 2$, as well as all points where $x = 2$. The set $D$ is not compact (closed, but not bounded).

**Problem 2.**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| a) No | b) No | c) No | d) Yes | e) No | f) Yes | g) Yes | h) Yes |
| i) No | j) No | k) No | l) No | m) Yes | n) Yes | o) No | p) No |

**Problem 4.**

a) $f_{\min} = 0$ in $(0,0)$, and $f_{\max} = 208$ in $(\pm 4, \pm 4)$

b) $f_{\min} = -15$ in $(0, \pm 4)$ and $(\pm 4, 0)$, and $f_{\max} = 225$ in $(\pm 4, \pm 4)$

c) $f_{\min} = 64$ in $(\pm 4, 0)$, and $f_{\max} = 144$ in $(0, \pm 4)$

d) $f_{\min} = 0$ in $(1,1)$ and $(-1,-1)$, and $f_{\max} = 225$ in $(4,4)$ and $(-4,-4)$

**Problem 5.**

a) $f_{\max} = 1,\ f_{\min} = -1$

b) $f_{\max} = 8,\ f_{\min} = -1$

c) $f_{\max} = 1,\ f_{\min} = 1/e^2$

d) $f_{\max} = 2\sqrt{3}/9,\ f_{\min} = -2\sqrt{3}/9$

e) $f_{\max} = 1,\ f_{\min} = 0$

**Problem 6.**

See the final exam of MET11807 06/2021 Exercise 5: $f_{\max} = 1/4,\ f_{\min} = -1$