Exercise session problems

Problem 1.
Compute the determinants:

a) $\begin{vmatrix} 3 & 7 \\ 1 & 3 \end{vmatrix}$ b) $\begin{vmatrix} 7 & 3 \\ 3 & 1 \end{vmatrix}$ c) $\begin{vmatrix} 3 & 1 \\ 7 & 3 \end{vmatrix}$ d) $\begin{vmatrix} 1 & 3 \\ 3 & 7 \end{vmatrix}$ e) $\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$ f) $\begin{vmatrix} a & b \\ b & c \end{vmatrix}$

Problem 2.
Compute the determinants:

a) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix}$ b) $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{vmatrix}$ c) $\begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & 5 \end{vmatrix}$ d) $\begin{vmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{vmatrix}$ e) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix}$

Problem 3.
Compute the determinants, and determine when they are zero:

a) $\begin{vmatrix} 2 & a \\ a & 8 \end{vmatrix}$ b) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & a \\ 1 & a & 9 \end{vmatrix}$ c) $\begin{vmatrix} a & 1 & 7 \\ 0 & 1-a & a \\ 0 & 0 & 2a \end{vmatrix}$ d) $\begin{vmatrix} 1 & 2 & a \\ 1 & a & 3 \\ 1 & a & 1 \end{vmatrix}$ e) $\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix}$

Problem 4.
Compute the determinants:

a) $\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix}$ b) $\begin{vmatrix} 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 2 \end{vmatrix}$ c) $\begin{vmatrix} 1 & 1 & 4 & 6 \\ 0 & 2 & \sqrt{3} & -1 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & -2 \end{vmatrix}$

Problem 5.
When $A = \begin{pmatrix} 3 & 7 \\ 4 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}$, we can think of the matrix

$$X = \begin{pmatrix} 3 & 7 & 0 & 0 \\ 4 & 6 & 0 & 0 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 3 & 1 \end{pmatrix}$$

as a *block-matrix* and write it as $X = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$, where each of the four blocks is a 2×2-matrix.

a) Compute $|X|$ b) Show that $|X| = |A| \cdot |B|$ c) Find $\begin{vmatrix} A & B \\ 0 & C \end{vmatrix}$ when C is a 2×2-matrix with $|C| = 4$

Problem 6.
We start with a quadratic matrix A, and end up with another matrix B by doing an elementary row operation. Will it always hold that $|A| = |B|$? Why/why not, and give examples.

Problem 7.
Determine when the following systems have exactly one solution, and use Cramer's rule to find the solutions in this case:

$$a)\quad \begin{array}{rcrcl} x & + & ay & = & 3 \\ ax & + & 4y & = & 1 \end{array} \qquad\qquad b)\quad \begin{array}{rcrcl} ax & + & y & = & 1 \\ -x & + & ay & = & 2 \end{array}$$

Problem 8.
A linear system is called *homogeneous* if all constant terms are zero. How many solutions does a homogeneous linear system with three equations and five unknowns have?

Problem 9.
Determine how many solutions the following linear systems have for different values of the parameter a.

$$a)\quad \begin{array}{rcrcrcl} x & + & 3y & + & az & = & 0 \\ 2x & - & ay & + & 3z & = & 0 \\ 3x & + & 2y & + & 4z & = & 0 \end{array} \qquad\qquad b)\quad \begin{array}{rcrcrcr} 2x & + & ay & - & z & = & a-5 \\ -x & + & 2y & + & az & = & -3 \\ ax & - & y & + & 2z & = & a+10 \end{array}$$

Problem 10.
Exam MET1180 (June 2016) Exercise 1abd
We consider the linear system of equations $A\mathbf{x} = \mathbf{b}$ given by

$$A = \begin{pmatrix} 2-s & 3 & 3 \\ 3 & 2-s & 3 \\ 3 & 3 & 2-s \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{og} \quad \mathbf{b} = \begin{pmatrix} 3 \\ s+4 \\ 1-2s \end{pmatrix}$$

View s as a parameter and x,y,z as variables.

a. **(6p)** Solve the linear system when $s = 8$. How many degrees of freedom does the system have?

b. **(6p)** Compute $|A|$ for an arbitrary value of s.

c. **(6p)** For which values of s does the linear system have exactly one solution? Find x in these cases.

Optional: Exercises from the Norwegian textbook

Textbook [E]:	Eriksen, *Matematikk for økonomi og finans*
Exercise book [O]:	Eriksen, *Matematikk for økonomi og finans - Oppgaver og Løsningsforslag*

Exercises:	[E] 6.3.1 - 6.3.7, 6.4.1 - 6.4.7
Solution manual:	Se [O] Kap 6.3 - 6.4
Exam exercises:	Se Exercise Sheet 32

Svar på veiledningsoppgaver

Problem 1.

a) 2 b) -2 c) 2 d) -2 e) 0 f) $ac - b^2$

Problem 2.

a) 2 b) 2 c) 0 d) 6 e) $(1-a)(1-b)(b-a)$

Problem 3.

a) Determinant $16 - a^2$, it is zero for $a = \pm 4$

b) Determinant $-a^2 + 2a + 7$, it is zero for $a = 1 \pm \sqrt{8}$

c) Determinant $2a^2(1-a)$, it is zero for $a = 0$ and $a = 1$

d) Determinant $4 - 2a$, it is zero for $a = 2$

e) Determinant $(a-1)^2(a+2)$, it is zero for $a = 1$ and $a = -2$

Problem 4.

a) 4 b) -10 c) -12

Problem 5.

a) 10 b) $10 = (-10) \cdot (-1)$ c) -40

Problem 6.

If we can go from A to B by adding a multiple of a row to another row, then $|A| = |B|$. If we exchange two rows, then $|B| = -|A|$. If we multiply a row with $c \neq 0$, then $|B| = c \cdot |A|$.

Problem 7.

a) $(x,y) = \left(\dfrac{12-a}{4-a^2}, \dfrac{1-3a}{4-a^2} \right)$ for $a \neq \pm 2$ b) $(x,y) = \left(\dfrac{a-2}{a^2+1}, \dfrac{2a+1}{a^2+1} \right)$ for all a

Problem 8.

Infinitely many solutions.

Problem 9.

a) Infinitely many solutions for $a = \pm 1$, one solution for $a \neq \pm 1$
b) Infinitely many solutions for $a = -1$, one solution for $a \neq -1$

Problem 10.

a) $(x,y,z) = (z-2, z-3, z)$, one degree of freedom (z er free)

b) $-s^3 + 6s^2 + 15s + 8 = -(s+1)^2(s-8)$

c) $s \neq 8, -1$, $x = 0$