

## Key Problems

### Problem 1.

Compute the indefinite integrals:

$$\text{a) } \int x^2 dx \qquad \text{b) } \int (8x^3 - 12x^2) dx \qquad \text{c) } \int (e^x - 6x) dx \qquad \text{d) } \int (x^2/3 - x^3/2) dx$$

### Problem 2.

Find a function  $f(x)$  with the given derivative and domain of definition:

$$\begin{array}{lll} \text{a) } f'(x) = 2, D_f = (-\infty, \infty) & \text{b) } f'(x) = 2x, D_f = (-\infty, \infty) & \text{c) } f'(x) = 6x^2, D_f = (-\infty, \infty) \\ \text{d) } f'(x) = 1/x, D_f = (0, \infty) & \text{e) } f'(x) = 1/x, D_f = (-\infty, 0) & \text{f) } f'(x) = 1/x, D_f = \{x : x \neq 0\} \end{array}$$

### Problem 3.

Find a function  $f(x)$  with the given properties:

$$\begin{array}{llll} \text{a) } \int f(x) dx = 2 + C & \text{b) } \int f(x) dx = 2x + C & \text{c) } \int f(x) dx = 6x^2 + C & \text{d) } \int f(x) dx = xe^{2x} + C \\ \text{e) } \int 2 dx = f(x) + C & \text{f) } \int 2x dx = f(x) + C & \text{g) } \int 6x^2 dx = f(x) + C & \text{h) } \int xe^{2x} dx = f(x) + C \end{array}$$

### Problem 4.

Compute the indefinite integrals:

$$\begin{array}{lllll} \text{a) } \int x^{-3} dx & \text{b) } \int \sqrt{x} dx & \text{c) } \int x\sqrt{x} dx & \text{d) } \int 1/x dx & \text{e) } \int 1/x^2 dx \\ \text{f) } \int (x - 2x^3) dx & \text{g) } \int x(1 - 2x) dx & \text{h) } \int x(1 - 2x) dx & \text{i) } \int (x + 1)^2 dx & \text{j) } \int (x + 1)^7 dx \end{array}$$

### Problem 5.

Compute the indefinite integrals:

$$\text{a) } \int \frac{1 - 3x^2}{x^2} dx \qquad \text{b) } \int \frac{x^3 + 2x - 2}{x} dx \qquad \text{c) } \int \frac{6x}{1 + 3x^2} dx \qquad \text{d) } \int \frac{\sqrt{x} + 1}{x^2} dx$$

### Problem 6.

Compute the indefinite integrals:

$$\text{a) } \int (1 + e^{2x}) dx \qquad \text{b) } \int e^{1+2x} dx \qquad \text{c) } \int e^{1-2x} dx \qquad \text{d) } \int 3^x dx$$

### Problem 7.

Compute the indefinite integrals:

$$\text{a) } \int x\sqrt{x^2 + 1} dx \qquad \text{b) } \int 9(x + 1)^7 dx \qquad \text{c) } \int xe^{-x^2} dx \qquad \text{d) } \int \frac{x}{1 + x^2} dx \qquad \text{e) } \int \frac{\ln x}{x} dx$$

**Problem 8.**

Compute the indefinite integrals:

a)  $\int \ln(x) \, dx$

b)  $\int \ln(x+1) \, dx$

c)  $\int \log_3(x) \, dx$

**Problem 9.**

Compute the indefinite integrals:

a)  $\int x \ln(x) \, dx$

b)  $\int x^2 \ln(x) \, dx$

c)  $\int \sqrt{x} \ln(x) \, dx$

d)  $\int x\sqrt{x} \ln(x) \, dx$

e)  $\int \frac{\ln(x)}{x^2} \, dx$

f)  $\int \frac{\ln(x)}{\sqrt{x}} \, dx$

g)  $\int \frac{\ln(x)}{x\sqrt{x}} \, dx$

h)  $\int \frac{\ln(x)}{x} \, dx$

**Problem 10.**

Compute the indefinite integrals:

a)  $\int xe^x \, dx$

b)  $\int xe^{-x} \, dx$

c)  $\int (x+1)e^x \, dx$

d)  $\int x^2e^x \, dx$

**Problem 11.**

Compute the indefinite integral:

$$\int \frac{e^{1-\sqrt{x}}}{\sqrt{x}} \, dx$$

**Problem 12.**

Assume that  $f(x) \geq 0$  for all  $x$ , and that  $F(x)$  is a function such that  $\int f(x) \, dx = F(x) + C$ . Is  $F(x)$  an increasing function? Explain why/why not.

**Problem 13.**

Problems from the textbook: 10.1.1 - 10.1.13

## Answers to Key Problems

**Problem 1.**

a)  $\frac{1}{3}x^3 + C$

b)  $2x^4 - 4x^3 + C$

c)  $e^x - 3x^2 + C$

d)  $\frac{1}{9}x^3 - \frac{1}{8}x^4 + C$

**Problem 2.**

a)  $f(x) = 2x$

b)  $f(x) = x^2$

c)  $f(x) = 2x^3$

d)  $f(x) = \ln(x)$

e)  $f(x) = \ln(-x)$  f)  $f(x) = \ln|x|$

**Problem 3.**

a)  $f(x) = 0$

b)  $f(x) = 2$

c)  $f(x) = 12x$

d)  $f(x) = (1 + 2x)e^{2x}$

e)  $f(x) = 2x$

f)  $f(x) = x^2$

g)  $f(x) = 2x^3$

h)  $f(x) = \left(\frac{1}{2}x - \frac{1}{4}\right)e^{2x}$

**Problem 4.**

- a)  $-\frac{1}{2}x^{-2} + \mathcal{C}$       b)  $\frac{2}{3}x\sqrt{x} + \mathcal{C}$       c)  $\frac{2}{5}x^2\sqrt{x} + \mathcal{C}$       d)  $\ln|x| + \mathcal{C}$   
 e)  $-1/x + \mathcal{C}$       f)  $\frac{1}{2}x^2 - \frac{1}{2}x^4 + \mathcal{C}$       g)  $\frac{1}{2}x^2 - \frac{2}{3}x^3 + \mathcal{C}$       h)  $x(x - x^2 + \mathcal{C})$   
 i)  $\frac{1}{3}(x+1)^3 + \mathcal{C}$       j)  $\frac{1}{8}(x+1)^8 + \mathcal{C}$

**Problem 5.**

- a)  $-1/x - 3x + \mathcal{C}$       b)  $\frac{1}{3}x^3 + 2x - 2\ln|x| + \mathcal{C}$       c)  $\ln(1 + 3x^2) + \mathcal{C}$       d)  $-2/\sqrt{x} - 1/x + \mathcal{C}$

**Problem 6.**

- a)  $x + \frac{1}{2}e^{2x} + \mathcal{C}$       b)  $\frac{1}{2}e^{1+2x} + \mathcal{C}$       c)  $-\frac{1}{2}e^{1-2x} + \mathcal{C}$       d)  $\frac{1}{\ln 3} \cdot 3^x + \mathcal{C}$

**Problem 7.**

- a)  $\frac{1}{3}(x^2 + 1)^{3/2} + \mathcal{C}$       b)  $\frac{9}{8}(x+1)^8 + \mathcal{C}$       c)  $-\frac{1}{2}e^{-x^2} + \mathcal{C}$       d)  $\frac{1}{2}\ln(1 + x^2) + \mathcal{C}$       e)  $\frac{1}{2}\ln(x)^2 + \mathcal{C}$

**Problem 8.**

- a)  $x \ln x - x + \mathcal{C}$       b)  $(x+1)\ln(x+1) - (x+1) + \mathcal{C}$       c)  $(x \ln x - x)/\ln(3) + \mathcal{C}$

**Problem 9.**

- a)  $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + \mathcal{C}$       b)  $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + \mathcal{C}$       c)  $\frac{2}{3}x\sqrt{x} \ln x - \frac{4}{9}x\sqrt{x} + \mathcal{C}$   
 d)  $\frac{2}{5}x^2\sqrt{x} \ln x - \frac{4}{25}x^2\sqrt{x} + \mathcal{C}$       e)  $-\frac{1}{x} \ln x - \frac{1}{x} + \mathcal{C}$       f)  $2\sqrt{x} \ln x - 4\sqrt{x} + \mathcal{C}$   
 g)  $-\frac{2}{\sqrt{x}} \ln x - \frac{4}{\sqrt{x}} + \mathcal{C}$       h)  $\frac{1}{2}(\ln x)^2 + \mathcal{C}$

**Problem 10.**

- a)  $xe^x - e^x + \mathcal{C}$       b)  $-xe^{-x} - e^{-x} + \mathcal{C}$       c)  $(x+1)e^x - e^x + \mathcal{C}$       d)  $x^2e^x - 2xe^x + 2e^x + \mathcal{C}$

**Problem 11.**

$$-2e^{1-\sqrt{x}} + \mathcal{C}$$

**Problem 12.**

Since  $F'(x) = f(x)$  and  $f(x) \geq 0$ , it follows that  $F$  is an increasing function.

**Problem 13.**

See answers in the textbook.