EBA1180 Mathematics for Data Science autumn 2024
Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

R. Lucas

Lecture 10
Sec. 4.1-6: Functions and graphs.
Linear and quadratic functions. Revenue and cost functions.

Here are recommended exercises from the textbook [SHSC].

Section **4.2** exercise 1-6, 13-15
Section **4.3** exercise 1-3
Section **4.4** exercise 1-10
Section **4.6** exercise 1-7

Problems for the exercise session
Wednesday 25 Sept. 12-16+ in CU1-067

Problem 1 Determine the expression of the linear function $f(x)$ in figure 1.

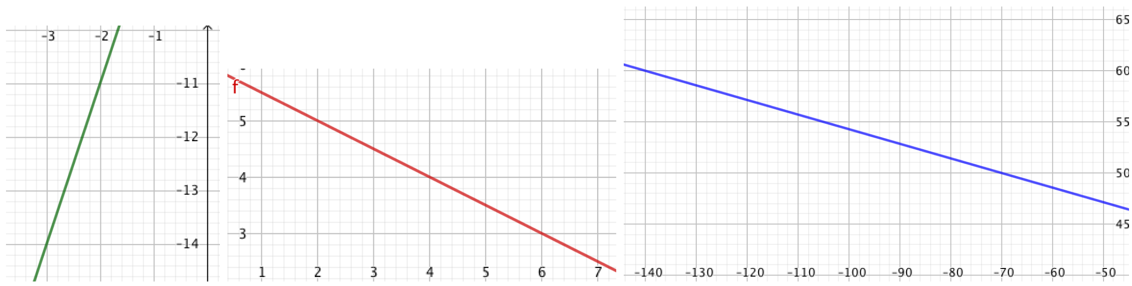

Figure 1: Linear functions (a-c)

Problem 2 Determine the intersection points of the graph and the x-axis and of the graph and the y-axis in Problem 1 a-c.

Problem 3 Determine the expression of the linear function $f(x)$ such that the graph passes through the points P and Q.

a) $P = (-2, 5)$ and $Q = (-4, 6)$

b) $P = (80, 90)$ and $Q = (50, 80)$

c) $P = (4, -3)$ and $Q = (-1, 7)$

Problem 4 Determine the expression of the linear function $f(x)$ such that the graph passes through the point P and has slope a.

a) $P = (-2, 5)$ and $a = \frac{2}{3}$

b) $P = (8, 90)$ and $a = \frac{1}{10}$

c) $P = (4, 30)$ and $a = -\frac{3}{10}$

Problem 5 Determine the expression of the second degree polynomial function $f(x)$ in a-f, see figure 2 and 3.

Problem 6 Determine the intersection points of the graph and the x-axis and of the graph and the y-axis in Problem 5 c-f.

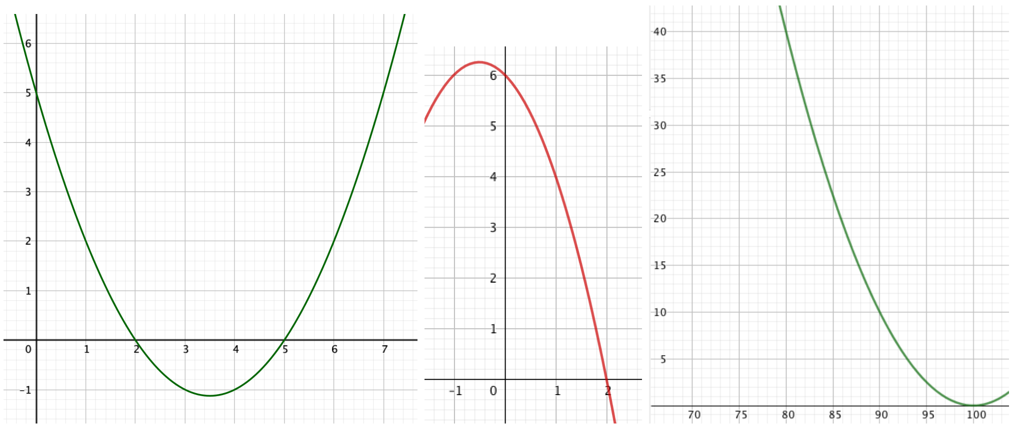

Figure 2: Parabolas (a-c)

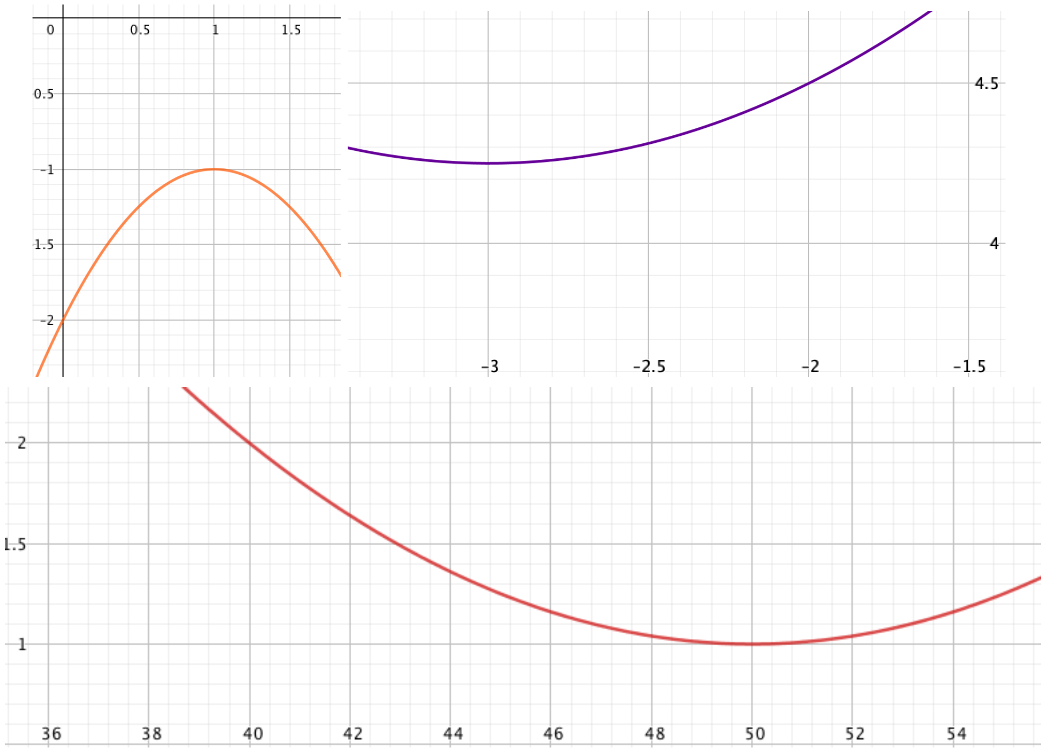

Figure 3: Parabolas (d-f)

Problem 7 Determine the expression of the second degree polynomial function $f(x)$ such that:

a) The graph passes through the points $P=(0,7)$, $Q=(1,4)$ and $R=(2,3)$.

b) The graph passes through the points $P=(-5,65)$, $Q=(3,65)$ and $R=(7,17)$.

c) The graph passes through the point $P=(4,-6)$ and $Q=(\frac{13}{2},-\frac{49}{4})$ is the point where the function attains its minimum.

Problem 8 Write $f(x)$ in the form $a(x-s)^2+d$ (by 'completing the square') and use the expression to sketch the graph. In particular determine the symmetry axis of the parabola and the maximum or minimum value.

a) $f(x)=x^2-10x+30$ b) $f(x)=3x^2+36x+110$ c) $f(x)=-\frac{1}{7}x^2+2x-6$

Problem 9 Suppose x is the number of units produced and sold. Determine the value of p (the unit price) which gives positive profit for $x>300$ and negative profit for $x<300$ if:

a) The cost function is $C(x)=2100+5x$ and the revenue function is $R(x)=px$.

b) The cost function is $C(x)=4500-5x+0.01x^2$ and the revenue function is $R(x)=px$ (both with $0\leqslant x\leqslant 1000$ as domain of definition).

Answers

Problem 1
a) $f(x) = 3x - 5$
b) $f(x) = -\frac{x}{2} + 6$
c) $f(x) = -\frac{x}{7} + 40$

Problem 2
a) $x = \frac{5}{3}$ and $y = -5$
b) $x = 12$ and $y = 6$
c) $x = 280$ and $y = 40$

Problem 3
a) $f(x) = -\frac{1}{2}x + 4$
b) $f(x) = \frac{1}{3}x + \frac{190}{3}$
c) $f(x) = -2x + 5$

Problem 4
a) $f(x) = \frac{2}{3}x + \frac{19}{3}$
b) $f(x) = \frac{1}{10}x + \frac{446}{5}$
c) $f(x) = -\frac{3}{10}x + \frac{156}{5}$

Problem 5
a) $f(x) = \frac{1}{2}(x-2)(x-5) = \frac{1}{2}x^2 - \frac{7}{2}x + 5$
b) $f(x) = -(x+3)(x-2) = -x^2 - x + 6$
c) $f(x) = \frac{1}{10}(x-100)^2 = \frac{1}{10}x^2 - 20x + 1000$
d) $f(x) = -(x-1)^2 - 1 = -x^2 + 2x - 2$
e) $f(x) = \frac{1}{4}(x+3)^2 + \frac{17}{4} = \frac{1}{4}x^2 + \frac{3}{2}x + \frac{13}{2}$
f) $f(x) = \frac{1}{100}(x-50)^2 + 1 = \frac{1}{100}x^2 - x + 26$

Problem 6
c) $x = 100$ and $y = 1000$
d) non and $y = -2$
e) non and $y = \frac{13}{2}$
f) non and $y = 26$

Problem 7
a) $(x-2)^2 + 3 = x^2 - 4x + 7$
b) $-(x+1)^2 + 81 = -x^2 - 2x + 80$
c) $(x - \frac{13}{2})^2 - \frac{49}{4} = x^2 - 13x + 30$

Problem 8
a) $f(x) = (x-5)^2 + 5$, the vertical line $x = 5$, $y = 5$ is the minimum value
b) $f(x) = 3(x+6)^2 + 2$, the vertical line $x = -6$, $y = 2$ is the minimum value
c) $f(x) = -\frac{1}{7}(x-7)^2 + 1$, the vertical line $x = 7$, $y = 1$ is the maximum value
For sketches of a-c see figure 4. A small table with relevant function values is expected.

Problem 9
a) $p = 3600/300 = 12$
b) $p = 13$

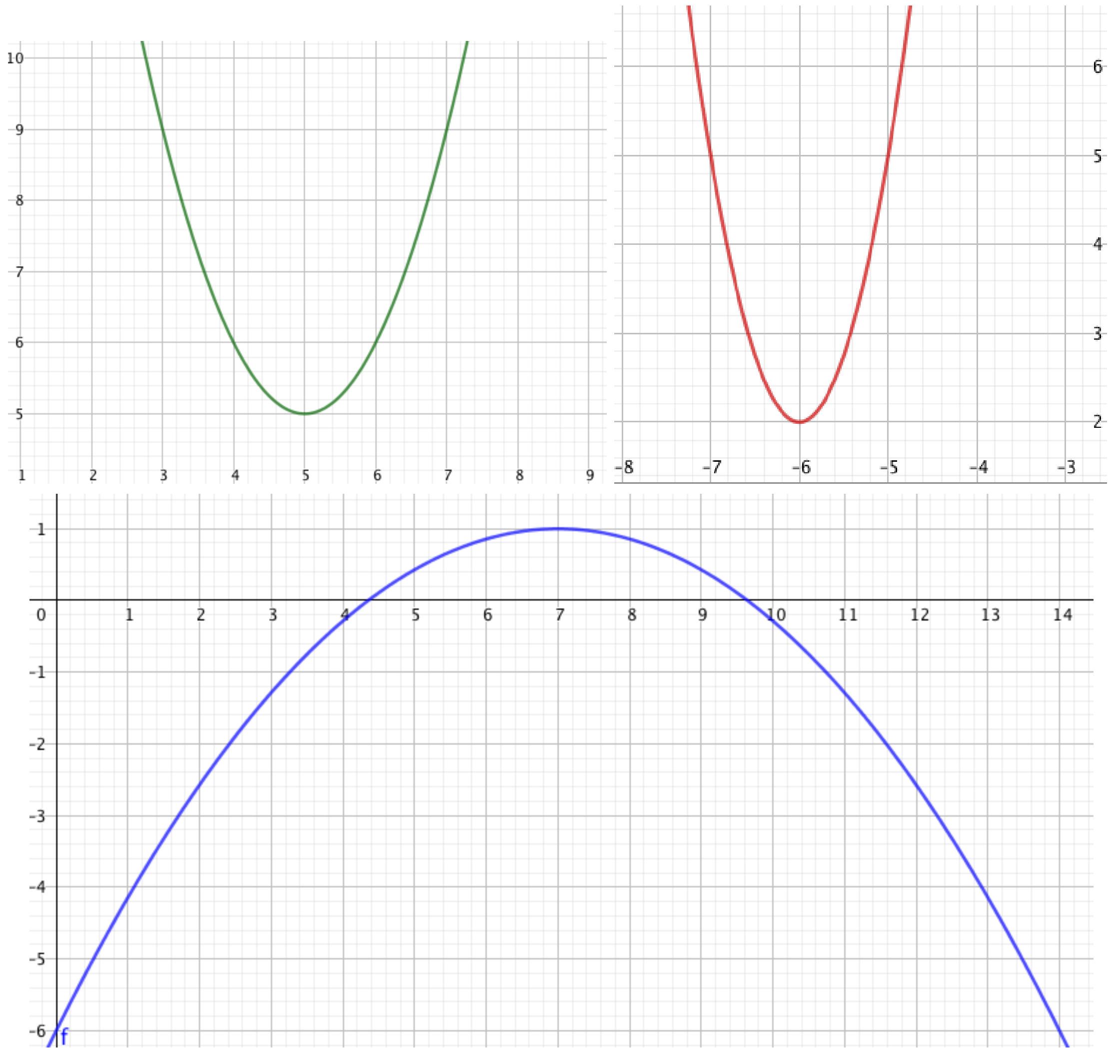

Figure 4: Parabolas in Problems 8 a-c