EBA 1180, lecture 9, 18 Sept 2024, Runar Ile

Plan
1. Rational equations
2. Radical equations
3. Inequalities

1. Rational equations

A rational eq.: $\dfrac{p(x)}{q(x)} = 0$ polynomials

Ex $\dfrac{x+1}{(x-1)(x+3)} = 0$ then $x+1=0$
and $(x-1)(x+3) \neq 0$
i.e. $x \neq 1$ and $x \neq -3$

so $\underline{\underline{x=-1}}$

Ex (Probl. 10a from last week). Solve the eq.

$(*)$ $1 + x + x^2 + \dots + x^{99} = 0$

Solution This is a geom. series with

$a_1 = 1$, $k = x$, number of terms $= 100$

The formula gives the LHS of the eq.:

$(**)$ $1 \cdot \dfrac{x^{100}-1}{x-1} = 0 \qquad (x \neq 1)$

Then $x^{100} - 1 = 0$ so $x^{100} = 1$

so $x = \pm 1^{\frac{1}{100}} = \pm 1$

So $x=-1$ is the only solution to $(**)$.

Have to check $x=1$ seperately in $(*)$:

LHS of $(*)$ is $1 + 1 + 1^2 + \dots + 1^{99} = 100 \neq 0$ (RHS)

so $\underline{\underline{x=-1}}$ is the only solution to $(*)$.

Ex $\quad \dfrac{x+1}{(x-1)(x+3)} \overset{(*)}{=} 2 \qquad \Big| -2$

$$\frac{x+1}{(x-1)(x+3)} - 2 = 0$$

Multiply -2 with $\dfrac{(x-1)(x+3)}{(x-1)(x+3)} = 1$

$$\frac{x+1-2(x-1)(x+3)}{(x-1)(x+3)} = 0$$

Resolve the parantheses in the numerator

$$\frac{x+1-2(x^2+2x-3)}{(x-1)(x+3)} = 0$$

Collect terms.

$$\frac{-2x^2-3x+7}{(x-1)(x+3)} = 0$$

that is: $-2x^2-3x+7=0$ and $x \neq 1$ and $x \neq -3$

which you can solve.

Note Could also multiply each side of $(*)$ with $(x-1)(x+3)$ - and remember that $x \neq 1$ and $x \neq -3$.

2. Radical equations

– the unknown (the x) is under a root

Ex $$2\sqrt{x+1} \overset{(*)}{=} x-2 \qquad (x \geq -1)$$

square both sides

$$4(x+1) = (x-2)^2 = (x-2)(x-2) = x^2-4x+4$$

$$4x+\cancel{4} = x^2-4x+\cancel{4}$$

$$x^2-8x=0$$

$$x(x-8)=0 \quad \text{so} \quad \underline{x=0} \text{ or } \underline{x=8}$$

Note Not all of these x-values need to be solutions of the original eq.

(think $-3 \neq 3$ but $(-3)^2 = 3^2$

We have to test the candidates:

$\underline{x=0}$ LHS: $2\cdot\sqrt{0+1} = 2\sqrt{1} = 2$

RHS: $0-2 = -2$

} not equal so $x=0$ is not a solution to $(*)$

$\underline{x=8}$ LHS: $2\cdot\sqrt{8+1} = 2\sqrt{9} = 6$

RHS: $8-2=6$

} – equal! so $\underline{\underline{x=8}}$ is the only solution to $(*)$.

3. Inequalities

$-2 < -1$ read: "minus two is less than minus one"

$\frac{1}{9} > \frac{1}{12}$ read: "one ninth is greater than one twelfth"

Also $\leqslant$ and $\geqslant$

- An inequality is a claim that one expression (number) is less than (greater than, ...) another expression (number)
- The solutions of an inequality are those values of x which make the claim true.

Start: 11.00

Ex $x-1 \geqslant 2$ is a claim.

- is true if $x = 5$ since $5-1=4 \geqslant 2$ is true
- in not true if $x = 2$ since $2-1=1 \geqslant 2$ is untrue

The solutions of the inequality are the values of x such that $\underline{\underline{x \geqslant 3}}$

Can also be written like this: $x \in [3, \rightarrow\rangle$ or

$x \in [3, \infty\rangle$

Ex Solve the inequality

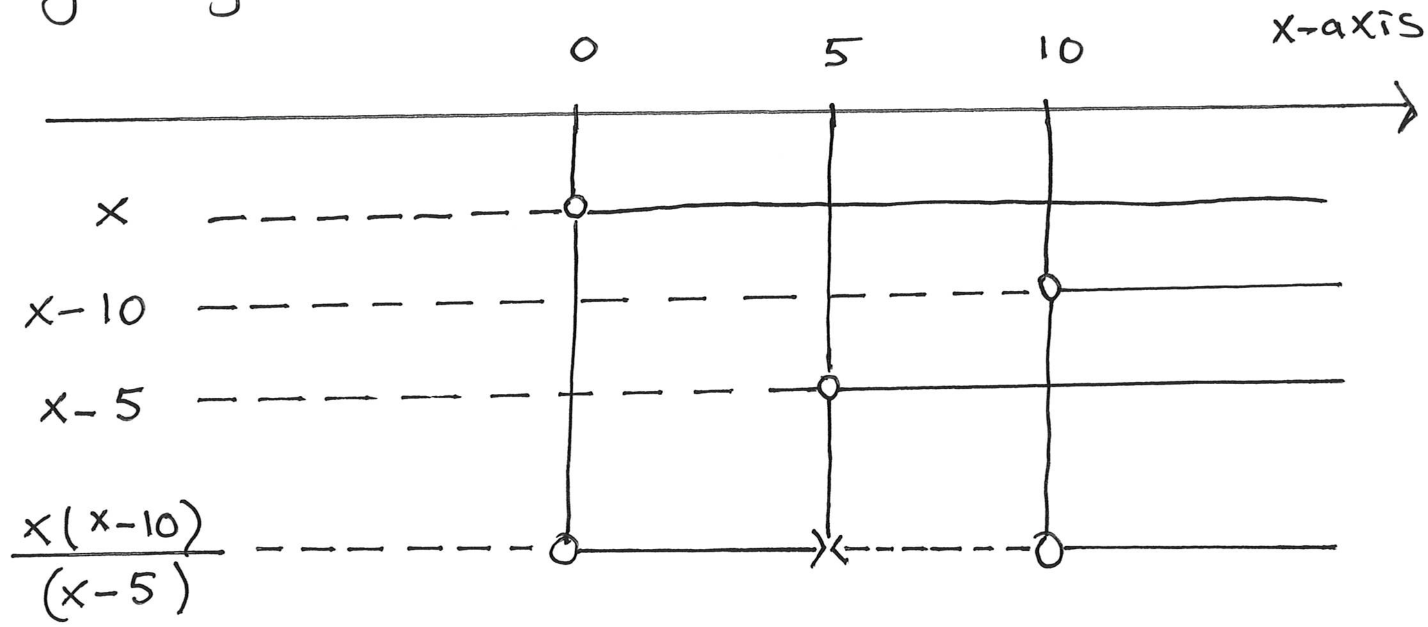
$$\frac{x(x-10)}{x-5} \geqslant 0$$

Solution Because we have 0 on the RHS and a factorised fraction on the LHS we can use a sign diagram directly:

Note zeros in the numerator: $x=0$, $x=10$

zeros in the denominator: $x=5$

Sign diagram:



that is $\underline{\underline{0 \leqslant x < 5 \quad \text{or} \quad x \geqslant 10}}$

We also write $\underline{\underline{x \in [0, 5\rangle \quad \text{or} \quad x \in [10, \infty\rangle}}$

EX $$\frac{2x-12}{(x-3)(x+4)} \geqslant 1 \quad \Big| -1$$ (Course Paper 2020a)

$$\frac{2x-12}{(x-3)(x+4)} - 1 \cdot \frac{(x-3)(x+4)}{(x-3)(x+4)} \geqslant 0$$

that is $$\frac{2x-12-(x-3)(x+4)}{(x-3)(x+4)} \geqslant 0$$

resolve and collect in the numerator

$$\frac{2x-12-(x^2+x-12)}{(x-3)(x+4)} \geq 0$$

$$\frac{-x^2+x}{(x-3)(x+4)} \geq 0$$

$$\frac{x(-x+1)}{(x-3)(x+4)} \geq 0$$

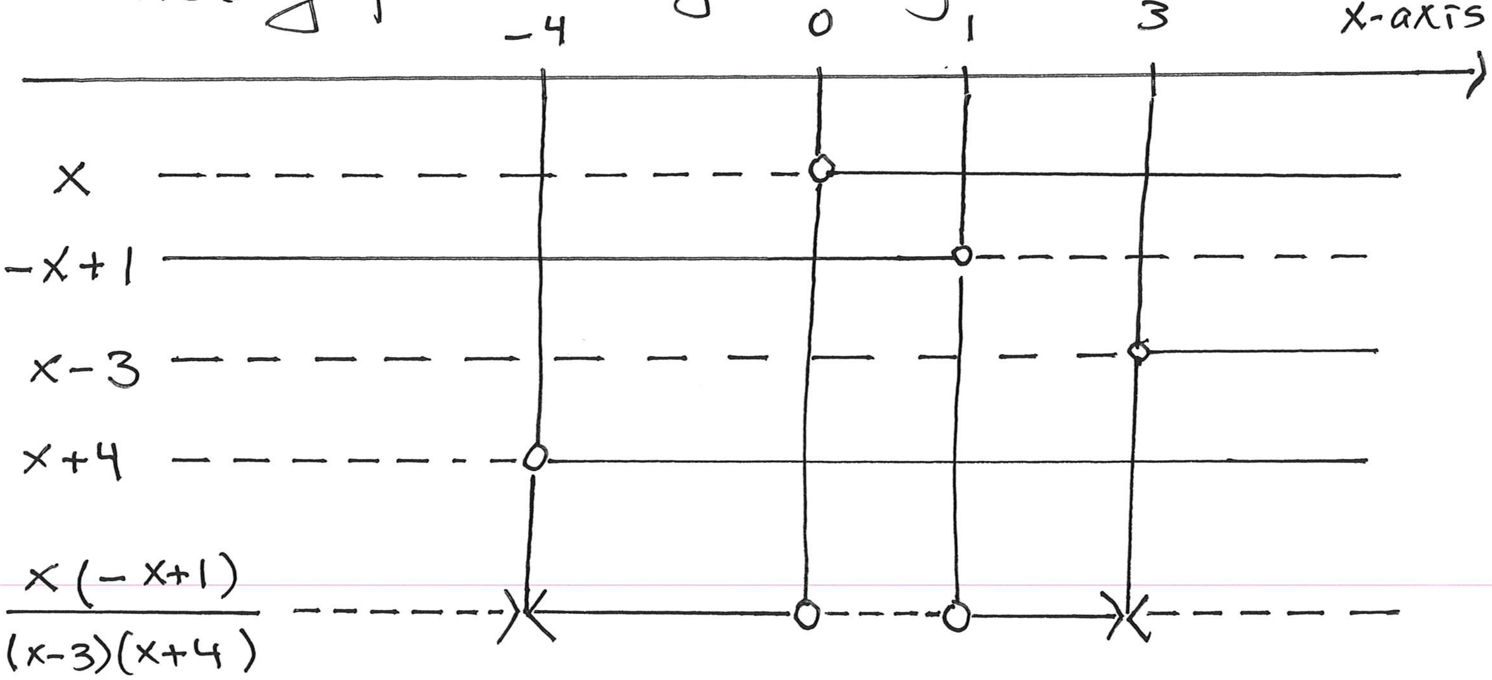
factorised LHS

zero on the RHS.

zeros in the numerator: $x=0$, $x=1$

zeros in the denominator: $x=3$, $x=-4$

– Ready for a sign diagram:



That is $\underline{\underline{-4 < x \leq 0 \quad \text{or} \quad 1 \leq x < 3}}$

Alternate way of writing: $\underline{\underline{x \in \langle -4, 0] \quad \text{or} \quad x \in [1, 3\rangle}}$