EBA 1180, lecture 6, 10 Sept. 2024, Rumar Ile

Plan
1. Repetition
2. Linear and quadratic equations.
3. Equations with parametres: the abc-formula

1. Repetition

Ex (Course Paper 2021a, probl. 1a)

i) Calculate the sum

$$6000 \cdot 1.0025^{96} + 6000 \cdot 1.0025^{95} + \ldots + 6000 \cdot 1.0025^{26} + 6000 \cdot 1.0025^{25}$$

Solution This is a geometric series with

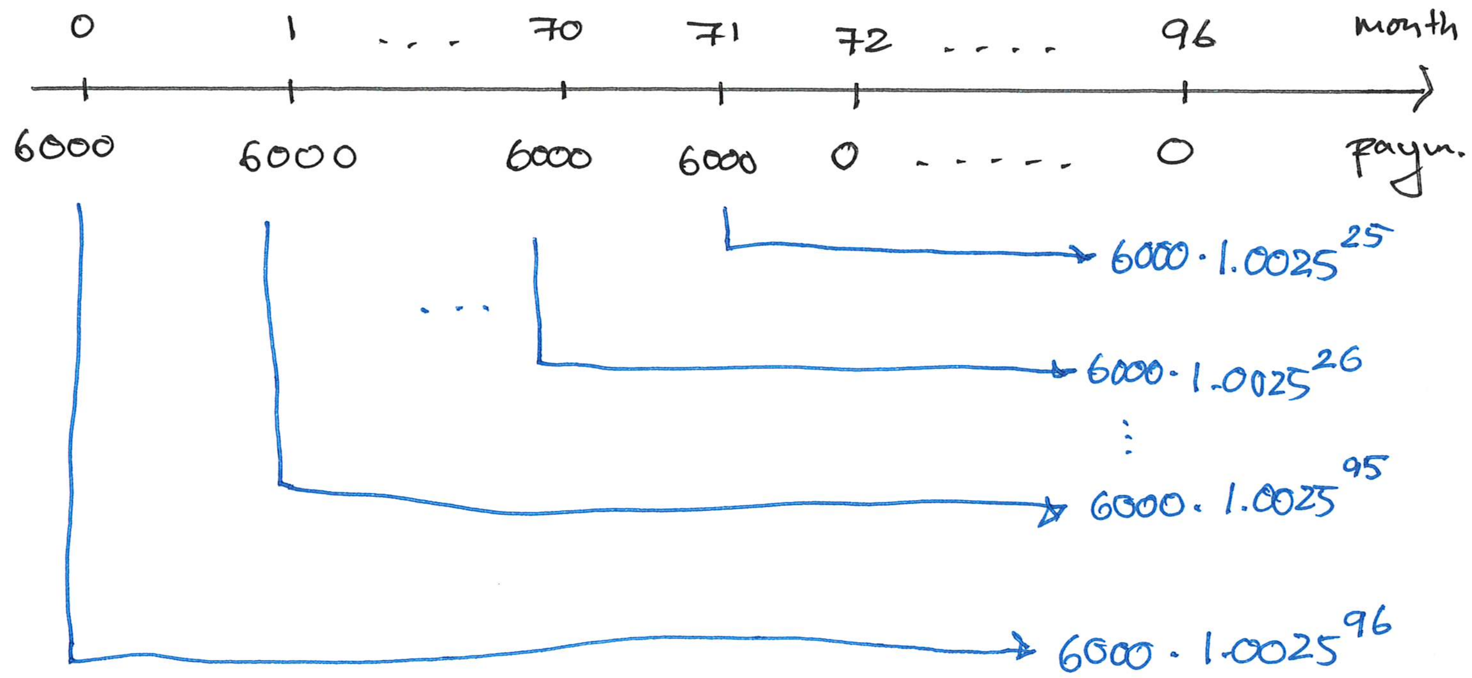
- the first term $= 6000 \cdot 1.0025^{25}$ (or: $6000 \cdot 1.0025^{96}$)
- the multiplicator $= 1.0025$ (or: $\frac{1}{1.0025} = 1.0025^{-1}$)
- the number of terms $= 96 - 24 = 72$ (no choice!)

The formula for the sum is

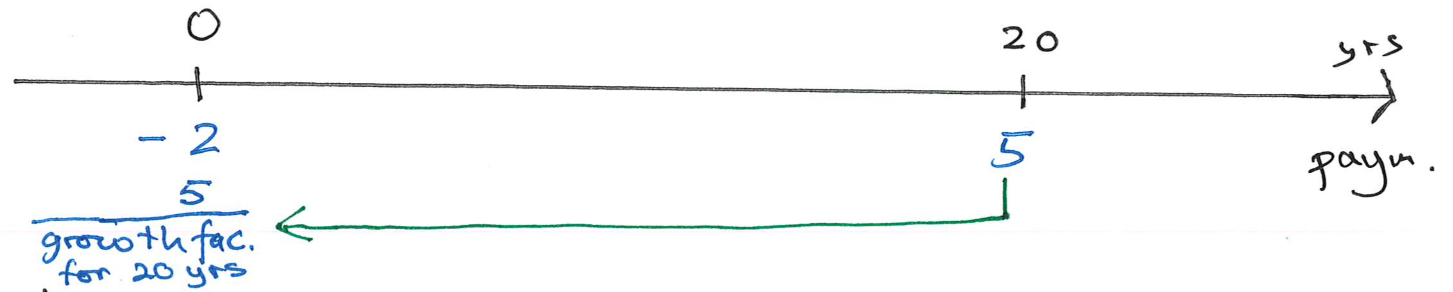
$$a_1 \cdot \frac{k^n - 1}{k - 1} = 6000 \cdot 1.0025^{25} \cdot \frac{1.0025^{72} - 1}{0.0025} = \underline{\underline{503\,122.08}}$$

ii) Describe a financial situation where the sum is relevant (the important number should be interpreted)

Solution The sum read from left to right gives the account balance after 8 years when 6000 is deposited every month for 6 years (so 72 deposits), with the first deposit today (the first term in the sum: $6000 \cdot 1.0025^{96}$), 3 % nominal interest, monthly compounding with periode rate $\frac{3\%}{12} = 0.0025$



Probl 5 (last week) You consider investing 2 mill today to receive 5 mill. 20 yrs. from now.



Q: What is the IRR? IRR (internal rate of return) is the interest that makes the tot. present value of the cash flow equal to 0. (Put r = IRR)

a) Annual compounding gives the eq.

$$-2 + \frac{5}{(1+r)^{20}} = 0 \quad \text{so} \quad \frac{5}{(1+r)^{20}} = 2 \;\Big|\cdot(1+r)^{20}$$

so $5 = 2\cdot(1+r)^{20} \;\Big|:2$ and get $(1+r)^{20} = \frac{5}{2}$

so $1+r = \sqrt[20]{\frac{5}{2}} = \left(\frac{5}{2}\right)^{\frac{1}{20}}$ and $r = \left(\frac{5}{2}\right)^{\frac{1}{20}} - 1$

$= \underline{\underline{4.69\,\%}}$

Calc: 2.5 $\boxed{y^x}$ 20 $\boxed{1/x}$ $\boxed{-}$ 1 $\boxed{=}$

b) With quaterly compounding there are $80 = 4 \cdot 20$ interest periods and r = quaterly IRR we get the eq.

$$\frac{5}{(1+r)^{80}} = 2$$

Then $r = \left(\frac{5}{2}\right)^{\frac{1}{80}} - 1 = 1.152\,\%$

and the nominal annual IRR is

$4 \cdot 1.152\,\% = \underline{\underline{4.61\,\%}}$

d) With continuous compounding the annual growth factor is e^r (r = nominal annual IRR).

so the tot. present value of the cash flow is $-2 + \frac{5}{(e^r)^{20}}$ which is supposed to be 0 which gives the eq. $\frac{5}{(e^r)^{20}} = 2 \;\Big|\, \cdot \frac{(e^r)^{20}}{2}$

so $(e^r)^{20} = \frac{5}{2}$ (or $e^{20r} = \frac{5}{2}$)

so $e^r = \left(\frac{5}{2}\right)^{\frac{1}{20}} = 1.0469$. We try different values of r (should be a little less than $4.61\,\%$)

A good answer is $r = \underline{\underline{4.58\%}}$

(or $r = \dfrac{\ln 5 - \ln 2}{20}$)

Start: 11.05

<u>Problem</u> You deposit 2 mill today, annual nominal interest is 12% with continuous compounding. Determine the balance after 1 yr. and 7 months.

<u>Solution</u> $$2\text{ mill}\cdot \underbrace{e^{0.12}}_{\substack{1\text{ yr.}\\ \text{growth f.}}} \cdot \overbrace{\Big(\underbrace{\left(e^{0.12}\right)^{\frac{1}{12}}}_{\substack{1\text{ month}\\ \text{growth f.}}}\Big)^{7}}^{7\text{ months growth f.}}$$

$$= 2\text{ mill}\cdot e^{0.12}\cdot\left(e^{0.01}\right)^{7}$$

$$= 2\text{ mill}\cdot e^{0.12}\cdot e^{0.07} = 2\text{ mill}\cdot e^{0.12+0.07}$$

$$= 2\text{ mill}\cdot e^{0.19} = \underline{\underline{2.42\text{ mill.}}}$$

2. Linear and quadratic equations

<u>A linear expression</u> $ax+b$ (a and b are numbers and $a \neq 0$)

<u>Ex</u> $4x-3$ $(a=4,\ b=-3)$

<u>A linear equation</u> An eq. which can be transformed into an equivalent eq. of the form $ax+b=0$ $(a\neq 0)$

Eq. The eq. $\frac{1}{x+3} = \frac{2}{x+4}$ $\quad x \neq -3,\ x \neq -4$ $\quad | \cdot (x+3)(x+4)$

multiply with a common denominator on each side.

$$x+4 = 2(x+3)$$

Use the distributive law on the RHS.

$$x+4 = 2x+6$$

Subtract 2x+6 on each side.

Std. form: $-x-2 = 0$ $\quad (a=-1,\ b=-2)$

$(x \neq -3,\ x \neq -4)$

A quadratic expression $\quad \underbrace{a}\,x^2 + \underbrace{b}\,x + \underbrace{c}_{\text{numbers}}$

A quadratic equation is an equation that can be made into the eq. $ax^2 + bx + c = 0$ $\quad (a \neq 0)$

$(x \neq 0,\ x \neq -1)$

Ex The eq. $\frac{1}{x} + \frac{2}{x+1} = 3$ is a quad. eq. because multiplying on each side with the common denominator $x(x+1)$ gives

$$x+1+2x = 3x(x+1)$$

resolve the paranthesis

$$3x + 1 = 3x^2 + 3x$$

subtract $3x^2 + 3x$ on each side

$$-3x^2 + 1 = 0 \quad (a = -3,\ b = 0,\ c = 1)$$

$$(x \neq 0,\ x \neq -1)$$

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3. Eq.s with parametres: The abc-formula

If $a \neq 0$ then $ax^2 + bx + c = 0$ has solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex $3x^2 + 4x - 5 = 0$ $(a = 3,\ b = 4,\ c = -5)$

Then $$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3}$$

$$= \frac{-4 \pm \sqrt{16 + 60}}{6} = \frac{-4 \pm \sqrt{76}}{6} \quad (76 = 4 \cdot 19)$$

$$= \frac{-4 \pm \sqrt{4} \cdot \sqrt{19}}{6} = \frac{\cancel{2}(-2 \pm \sqrt{19})}{\cancel{2} \cdot 3}$$

$$= \underline{\underline{\frac{-2 \pm \sqrt{19}}{3}}} = \underline{\underline{-\frac{2}{3} \pm \frac{\sqrt{19}}{3}}}$$