EBA 1180, lecture 3, 28 Aug. 2024, Runar Ile

Plan 1. A few examples
2. Total present value of a cash flow and IRR (internal rate of return).

1. A few examples

Problem The value of Kåre's flat increases by 10 % the first year, and decreases by 30 % the second year. Compute the relative change for the two years combined.
(Hint: Answer is not −20%)

Solution
Relative change for the first year: $r_1 = 0.1$
—— " —— second —"—: $r_2 = -0.3$

Rate of change for the first year: $1+r_1 = 1.1$
—— " —— second —"—: $1+r_2 = 0.7$
—— " —— two years combined:
$(1+r_1)\cdot(1+r_2) = 1.1 \cdot 0.7 = 0.77$

So the relative change for the two years is $0.77 - 1 = -0.23 = \underline{\underline{-23\ \%}}$

Pattern Relative changes of value: $r_1, r_2, \dots, r_n$ gives the combined relative change

$$\underbrace{(1+r_1)\cdot(1+r_2)\cdot\ldots\cdot(1+r_n)}_{\text{combined rate of change}} - 1$$

Ex Deposit (principal) : 50 000

Interest : $r = 4\%$ with annual compounding

After 5 years the balance is

$$50\,000 \cdot (1 + 4\%)^5 = \underline{\underline{60\,832.65}}$$

Calculator : 50 000 $\boxed{\times}$ 1.04 $\boxed{y^x}$ $\boxed{5}$ $\boxed{=}$

Problem Deposit : 50 000

Nominal interest: 4 %

Monthly compounding.

a) Determine the balance after 5 years.

b) Determine the effective interest.

Solution a) After 5 years the balance is

$$50\,000 \cdot \left(1 + \frac{4\%}{12}\right)^{12 \cdot 5}$$

$$= 50\,000 \cdot \left(1 + \frac{0.04}{12}\right)^{60} = \underline{\underline{61\,049.83}}$$

b) Effective interest r_{eff} = the annual interest which gives the same balance as the period rate (relative change for one year)

$$1 + r_{eff} = \left(1 + \frac{0.04}{12}\right)^{12}$$

$$= 1.040742$$

so $r_{eff} = 0.040742 = \underline{\underline{4.0742\,\%}}$

Problem After 5 years of added interest the deposit of 50 000 has become 60 000. Calculate the effective interest.

Start : 11.00

Solution The 5-year growth factor (rate of change) is

$$\frac{60\,000}{50\,000} = 1.2$$

Let g be the annual growth factor.

Then $g^5 = 1.2$

A bit slower:

balance (years)

$$\begin{array}{ccccccc} 0 & & 1 & & 2 \;\cdots & & 5 \\ 50' & \xrightarrow{\cdot g} & 50' g & \xrightarrow{\cdot g} & 50' \cdot g^2 & \xrightarrow{\cdot g^3} & 50' \cdot g^5 \end{array}$$

$\overset{\text{by assumption}}{=} 60'$

Then $50\,000 \cdot g^5 = 60\,000 \quad | : 50\,000$

$$g^5 = \frac{60\,000}{50\,000} = 1.2$$

so the annual growth factor

$$g = (g^5)^{\frac{1}{5}} = 1.2^{\frac{1}{5}} \qquad \left(= \sqrt[5]{1.2}\right)$$

$$g = 1.2^{0.2} = 1.03714$$

so $r_{eff} = 0.03714 = \underline{\underline{3.714\,\%}}$

2. Total present value of a cash flow and IRR

Present value of an amount (K) paid n years from now with interest r

= what you have to deposit today (K_0) for the balance to be K n years from now with interest r.

Since $K_0 (1+r)^n = K \quad | : (1+r)^n$

$$K_0 = \frac{K}{(1+r)^n} \quad \text{(present value)}$$

Ex 50000 (K) 3 years from now with 4% interest has present value

$$K_0 = \frac{50\,000}{(1.04)^3} = \underline{\underline{44\,449.82}}$$

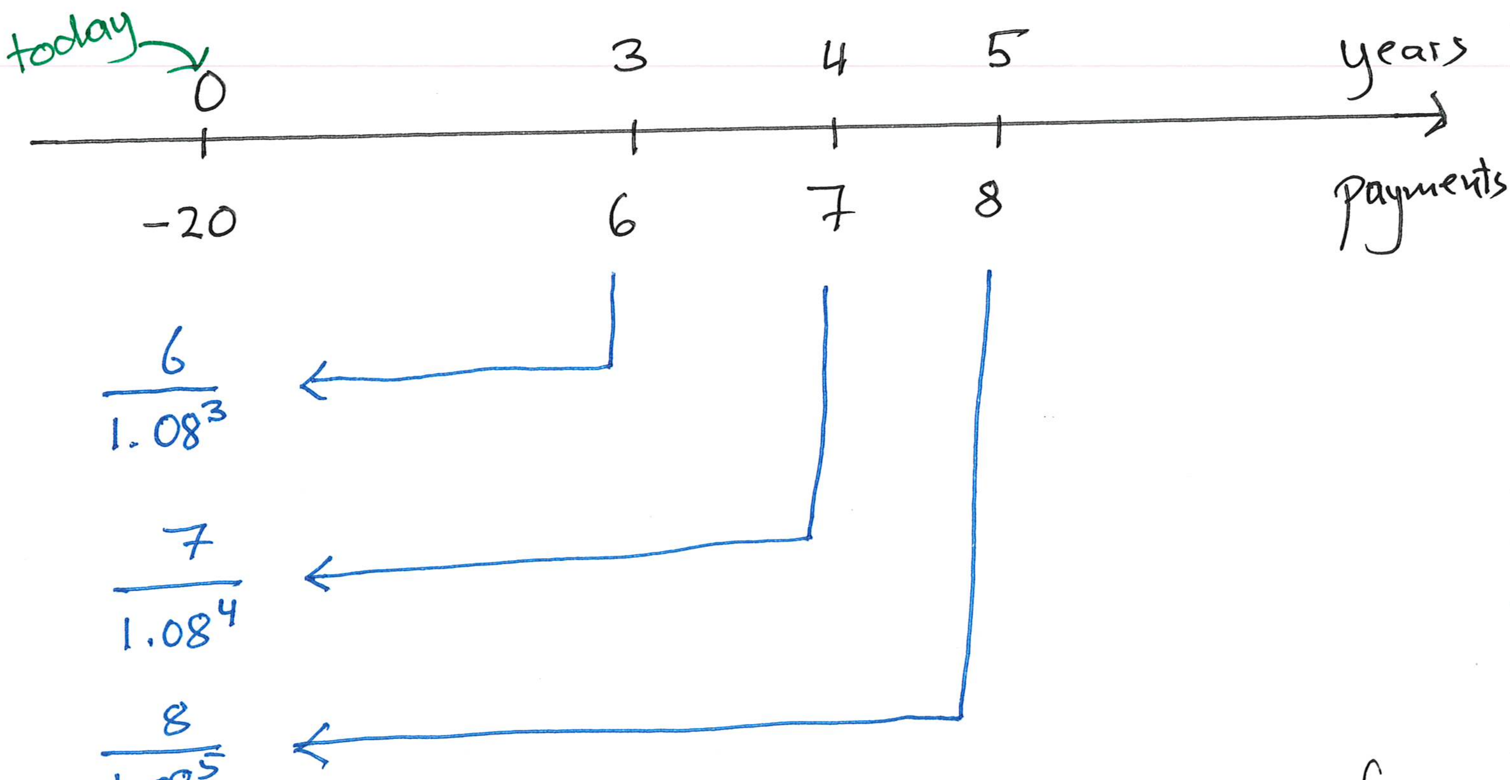
Calc: 1.04 $\boxed{y^x}$ 3 $\boxed{=}$ $\boxed{1/x}$ $\boxed{\times}$ 50 000 $\boxed{=}$

or 50 000 $\boxed{\div}$ 1.04 $\boxed{y^x}$ 3 $\boxed{=}$

We can do this for a cash flow (several payments combined)

Ex You pay 20 mill today, and get back
6 mill after 3 years
7 — " — 4 — " —
8 — " — 5 — " —

With 8 % interest, what is the total present value of the cash flow?
It is the sum of the present values of each of the payments.



The sum = tot. present value of the cash flow

$$= -20 + \frac{6}{1.08^3} + \frac{7}{1.08^4} + \frac{8}{1.08^5} = \underline{\underline{-4.65}}$$

Negative tot. pres. val. means investment is not giving 8% annual yield.

With these back payments, the borrower can borrow 15.35 mill (not 20).

Or: The borrower can pay $4.65 \cdot 1.08^6$ extra after 6 years (an example)

In both these alternative cases the totat present value of the cash flow is 0.

The internal rate of return (IRR) of a given cash flow is the interest which makes the tot. pres. value of the cash flow equal to 0.

In general hard to find IRR by hand.

Here we have to solve the equation

$$f(x) = -20 + \frac{6}{(1+x)^3} + \frac{7}{(1+x)^4} + \frac{8}{(1+x)^5} = 0$$

(x = interest)

Answer: $x \approx 1.12\%$