Recap
Integration methods

- Substitution: $u = g(x)$

$$du = g'(x)\,dx$$

$$dx = \frac{1}{g'(x)}\,du$$

- Integration by parts:

$$\int u' \cdot v \, dx = uv - \int u v' \, dx$$

EBA1180 Lect. 3 S25 → 27

Thursday: Problem set 27 (old exams) ⤳ Try yourself before class

Its learning

Q: Plan?

1) $\int 5x^2 \cdot e^x \, dx$ → Int. by parts 2 times

2) $\int 5\,x^2 \cdot e^{x^3} \, dx$ → Substitute $u = x^3$, $du = 3x^2 dx$

$$\int u' v \, dx = uv - \int u v' \, dx$$

Ex: $\int x \cdot \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}\cancel{x^2}\,\cancel{\frac{1}{x}}\, dx$

Int. by parts:

$v = \ln x \Rightarrow v' = \frac{1}{x}$

$u' = x \Rightarrow u = \frac{1}{2}x^2$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\,\frac{1}{2}x^2 + C$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

Ex: $\int 2x\cdot e^{x}\,dx = e^{x}2x - \int e^{x}\cdot 2\,dx$

$\int v u'\,dx = uv - \int u v'\,dx$

Int. by parts:

$u' = e^{x} \Rightarrow u = e^{x}$

$v = 2x \Rightarrow v' = 2$

$= 2xe^{x} - 2\int e^{x}\,dx$

$= 2xe^{x} - 2e^{x} + C$

TO CHECK: Differentiate your answer and see that you get the integrand back

Does it work to choose opposite roles?

$\int u' v\,dx = uv - \int u v'\,dx$

$\int 2xe^{x}\,dx = x^{2}e^{x} - \int x^{2}e^{x}\,dx$

Int. by parts:

$u' = 2x \Rightarrow u = x^{2}$

$v = e^{x} \Rightarrow v' = e^{x}$

Looks worse! Doesn't work well with opposite roles.

Ex: $\int \ln x\,dx = \int 1\cdot \ln x\,dx$

$\int u' v\,dx$

TRICK

$uv - \int u v'\,dx$

$= x\ln x - \int \cancel{x}\frac{1}{\cancel{x}}\,dx$

Int. by parts:

$u' = 1 \Rightarrow u = x$

$v = \ln x \Rightarrow v' = \frac{1}{x}$

$= x\ln x - \int 1\,dx$

$= x\ln x - x + C$

FORMULA: $\boxed{\int \ln x \, dx = x \ln x - x + C}$

$$\int v u' dx = uv - \int v' u \, dx$$

Ex: $\int x^2 e^x dx = e^x x^2 - \underbrace{\int 2x e^x dx}$

Int. by parts:
$u' = e^x \Rightarrow u = e^x$
$v = x^2 \Rightarrow v' = 2x$

Int. by parts: Did earlier today!

x^2 leads to 2* int. by parts $\rightarrow x^n$ leads to n* int. by parts

$$= x^2 e^x - (2xe^x - 2e^x) + C$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

START 11.00

METHOD: Integration of rational functions

Ex: i) $\int \frac{2}{1-x} dx$ ii) $\int \frac{2x}{1-x^2} dx$

iii) $\int \frac{2}{1-x^2} dx$

Q: Discuss and guess: Which technique? Plan?

What to do?

i) $\int \frac{2}{1-x} dx = \int \frac{2}{u} (-du)$

SUBSTITUTION:
$u = 1 - x$
$du = -dx$
$dx = -du$

$$= -2 \int \frac{1}{u} du$$

$$= -2 \ln|u| + C$$

$$= -2\ln|1-x| + C$$

In general: If $a \neq 0$:

$$\int \frac{A}{ax+b}\,dx = \int \frac{A}{u}\,\frac{1}{a}\,du$$

SUBSTITUTION:

$u = ax+b$

$du = a\,dx$

$dx = \frac{1}{a}\,du$

$$= \frac{A}{a}\int \frac{1}{u}\,du$$

$$= \frac{A}{a}\ln|u| + C$$

$$= \frac{A}{a}\ln|ax+b| + C$$

FORMULA:

$$\boxed{\int \frac{A}{ax+b}\,dx = \frac{A}{a}\ln|ax+b| + C, \quad a \neq 0}$$

Ex (alt.): $\displaystyle\int \frac{x}{1-x}\,dx = \int -1 + \frac{1}{1-x}\,dx$

$$= -x - \ln|1-x| + C$$

Polynomial div:
If
degree numerator $\geq$ degree denominator

If you can do a polynomial division: Do.

Polynomial division:

$$x : (-x+1) = -1 + \frac{1}{1-x}$$

$$\frac{-(x-1)}{1}$$

Remainder

Type ii) : $\int \frac{2x}{1-x^2}\,dx = \int \frac{\cancel{2x}}{u}\left(-\frac{1}{\cancel{2x}}\right)du$

SUBSTITUTION:

$u = 1 - x^2$

$du = -2x\,dx$

$dx = -\frac{1}{2x}\,du$

$= -\int \frac{1}{u}\,du$

$= -\ln|u| + C$

$= \underline{\underline{-\ln|1-x^2| + C}}$

Type iii) : $\int \frac{2}{1-x^2}\,dx = \int \frac{\cancel{2}}{u}\,\frac{1}{\cancel{-2}x}\,du$

SUBSTITUTION?

$u = 1 - x^2$

$du = -2x\,dx$

$dx = -\frac{1}{2x}\,du$

$= \underbrace{-\int \frac{1}{xu}\,du}_{\text{Looks bad!}}$

Actually: Not solvable by substitution. Instead:

METHOD: Partial fractions → Delbrøks-oppspaltning

Ex: $\frac{2}{1-x^2} = \frac{A}{1+x} + \frac{B}{1-x}$

AIM: Find A and B such that

Factorize denominator:

$1 - x^2 = (1-x)(1+x)$

NB: A and B are unknown constants.

$$2 = \frac{A}{\cancel{1+x}}(1-x)\cancel{(1+x)} + \frac{B}{\cancel{1-x}}\cancel{(1-x)}(1+x)$$

$$2 = A(1-x) + B(1+x)$$

$$2 = A - Ax + B + Bx$$

$$0\cdot x + 2 = (B-A)x + (A+B) \qquad (\star)$$

compare coefficients

1) $B - A = 0 \Rightarrow B = A$

2) $A + B = 2$

$$2A = 2$$

$$A = 1 \Rightarrow B = 1$$

Hence, $$\frac{2}{1-x^2} = \frac{A}{1+x} + \frac{B}{1-x} = \frac{1}{1+x} + \frac{1}{1-x}$$

Why OK to "compare coefficients"?

Eq. $(\star)$ holds for all x, e.g. $x=0$:

$$\boxed{2 = A+B}$$

$$0\cdot x + \cancel{2} = (B-A)x + \underbrace{\cancel{(A+B)}}^{2} \qquad (\star)$$

$$0\cdot x = (B-A)x$$

Say $x=1$: $\quad 0\cdot 1 = (B-A)\cdot 1$

$$\boxed{0 = B - A}$$

DO PROBLEM SET 2.7 AT HOME BEFORE THURSDAY!