EBA1180, lecture 2, 27 Aug. 2024, Runar Ile

Plan 1. Repetition (alg. exp., roots & powers, absolute value)
2. Relative change and rate of change
3. Interest
4. Present value

1. Repetition

Fractions $\frac{2}{3}\cdot\frac{5}{4} = \frac{2\cdot 5}{3\cdot 4} = \frac{10}{12}$

and $\frac{x+3}{x+4}\cdot\frac{x-1}{x+2} = \frac{(x+3)\cdot(x-1)}{(x+4)\cdot(x+2)}$

Probl. 1i $\frac{18}{4}\cdot\frac{\frac{2}{3}}{12} = \frac{18\cdot\frac{2}{3}}{4\cdot 12} = \frac{\frac{18}{1}\cdot\frac{2}{3}}{4\cdot 12} = \frac{\frac{18\cdot 2}{1\cdot 3}}{4\cdot 12}$

$= \frac{\frac{18\cdot 2}{3}\cdot 3}{4\cdot 12\cdot 3} = \frac{\frac{18\cdot 2\cdot\cancel{3}}{\cancel{3}}}{4\cdot 12\cdot 3} = \frac{18\cdot 2}{4\cdot 12\cdot 3} = \frac{\cancel{2}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{2}}{\cancel{2}\cdot\cancel{2}\cdot\cancel{3}\cdot 2\cdot 2\cdot\cancel{3}}$

$= \frac{1}{4}$

Prob 2i $\frac{x^2-3x}{x(y-3)}\cdot\frac{xy^2-9x}{x-3} = \frac{\cancel{x}(x-3)}{\cancel{x}(y-3)}\cdot\frac{x(y^2-9)}{x-3}$

$= \frac{x-3}{y-3}\cdot\frac{x(y-3)(y+3)}{x-3} = \frac{\overset{1}{\cancel{(x-3)}}\cdot x\cdot\overset{1}{\cancel{(y-3)}}\cdot(y+3)}{\underset{1}{\cancel{(y-3)}}\cdot\underset{1}{\cancel{(x-3)}}}$

$= \frac{x(y+3)}{1} = x(y+3)$.

Order of operations

$2+3\cdot 4 = 14$

$(2+3)\cdot 4 = 20$

$-3^2 = (-1)\cdot 3\cdot 3 = -9$

$(-3)^2 = (-3)\cdot(-3) = 9$

$-3\cdot 4 = -12$

$-1^2 = (-1)\cdot 1\cdot 1 = -1$

Roots/Powers

$\sqrt{5} = 5^{0,5} = 5^{\frac{1}{2}}$

$\sqrt{5} \cdot \sqrt{5} = 5^{0,5} \cdot 5^{0,5} = 5^{0.5+0.5} = 5^1 = 5$

$\sqrt[3]{5} = 5^{\frac{1}{3}}$ so $(\sqrt[3]{5})^6 = (5^{\frac{1}{3}})^6 = 5^{\frac{1}{3}\cdot 6} = 5^2$

Moreover: $5^{-1} = \frac{1}{5}$ and $5^{-2} = \frac{1}{5^2}$

<u>Pattern</u> If m, n integers, $n > 0$ and $a > 0$ (a pos. number), then

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

<u>Probl 6l</u> $\frac{\sqrt{1.03}^{10}}{1.03^4} = \frac{(1.03^{\frac{1}{2}})^{10}}{1.03^4} = \frac{1.03^5}{1.03^4}$

$= 1.03^{5-4} = \underline{\underline{1.03}}$

<u>Problem</u> Calculate $1.11^{\sqrt{2}}$ on your calculator (answer: 1.159035.....)

<u>solution</u> 1.11 $\boxed{y^x}$ 2 $\boxed{\sqrt{x}}$ $\boxed{=}$

Same base: $2^{1.5} \cdot 2^{3.8} = 2^{1.5+3.8} = 2^{5.3}$

Same exponent: $2^4 \cdot 3^4 = (2\cdot 3)^4 = 6^4$

<u>Ex</u> $\sqrt{2} \cdot \sqrt{3} = 2^{½} \cdot 3^{½} = (2\cdot 3)^{½} = \sqrt{6}$

<u>Pattern</u> $a^r \cdot b^r = (ab)^r$

Probl Calc. 1.12^{-1} on your calc.

Solution 1 $\quad 1.12 \ \boxed{y^x} \ 1 \ \boxed{+/-} \ \boxed{=}$

Solution 2 $\quad 1.12 \ \boxed{1/x} \quad$ (reason: $1.12^{-1} = \frac{1}{1.12}$)

Absolute value

Ex $\sqrt{(-3)^2} = \sqrt{(-3)\cdot(-3)} = \sqrt{9} = 3 = -(-3) = |-3|$

so $\sqrt{x^2} = |x|$

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Start: 11.00

2. Relative change and rate of change

$$\text{Relative change} = \frac{\text{new value} - \text{old value}}{\text{old value}}$$

Recall $\% = \frac{1}{100} = 0.01$

$3\% = 3 \cdot \frac{1}{100} = 0.03$

Ex Kåre's hourly wage increased from 163 kr to 181 kr. The relative change was

$$\frac{181\,\text{kr} - 163\,\text{kr}}{163\,\text{kr}} = \frac{18}{163} = 11.0\,\%$$

$$\boxed{\begin{aligned}\text{Rate of change} &= 1 + \text{relative change}\\ &= \frac{\text{new value}}{\text{old value}}\end{aligned}}$$

Ex The rate of change in Kåre's hourly wage is $1 + 0.11 = 1.11$

Probl Last year Kåre earned 54 000 kr with 163 kr/hour. If he works as much this year as last year, how much will he earn? (with the new wage)

Solution $54000 \cdot 1.11 = \underline{\underline{59\,940}}$

3. Interest

Ex You deposit 40 000 into an account earning 2.3 % annual interest. Interest is added each year (annual compounding of interest)

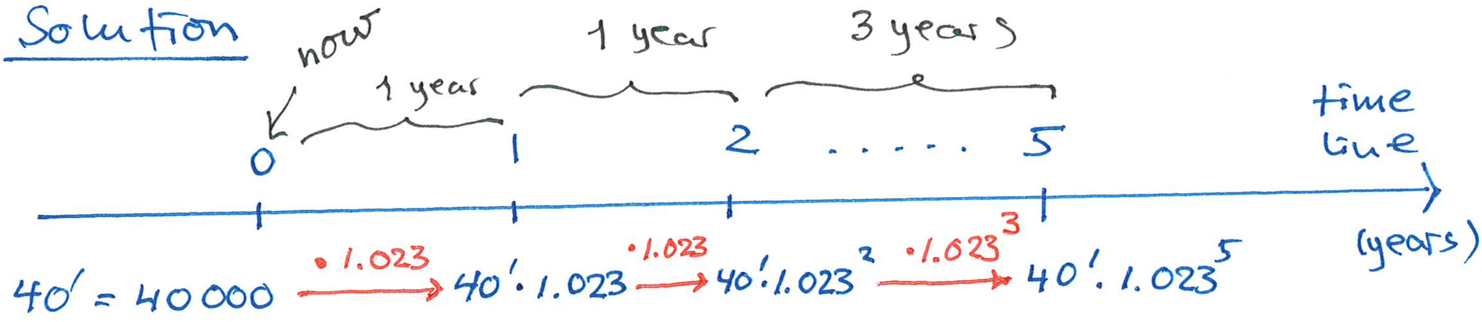
After a year the balance (what's in the account) is

$$40\,000 + 40\,000 \cdot 2.3\%$$

$$= 40\,000 \cdot \underbrace{(1 + 2.3\%)}_{\substack{\text{rate of change,}\\ \text{the growth factor}}} = \underline{\underline{40\,920.00}}$$

Probl What is the balance after 5 years?

Solution



$$\underline{\underline{40\,000 \cdot 1.023^5}} = \underline{\underline{44816.52}}$$

Ex You deposit 40 000 with 2.3% nominal annual interest, but with quaterly compounding of interest. The growth factor (rate of change) for one period (= 3 months) is

$$1 + \frac{2.3\%}{4} = 1 + \underbrace{0.575\%}_{\text{interest for one period}} = 1.00575$$

After 1 year the balance is

$$40\,000 \cdot 1.00575^4 = 40\,927.96$$

The annual growth factor (rate of change) is $1.00575^4 = 1.023199$

The effective (annual) interest is

$$1.00575^4 - 1 = 0.023199 = 2.3199\%$$

nominal interest

Pattern $B = B_0 \cdot \left(1 + \frac{r}{n}\right)^{m}$ number of periods

balance after m periods

deposit (principal)

number of interest periods in a year

4. Present value

Let K_0 be some investment/deposit/payment today. The future value K_n of K_0 in n years (or more generally, n periods) with interest r is

$$K_n = K_0 \cdot (1+r)^n$$

The opposite: suppose K_n will be paid n years (periods) from now, with annual (period) interest r. Then the present value K_0 of K_n is given as

$$K_0 = \frac{K_n}{(1+r)^n}$$

Ex 30 mill. is paid 5 years from now with 8% (annual) interest.

The present value is $K_0 = \frac{30 \text{ mill}}{1.08^5} = \underline{\underline{20.42 \text{ mill}}}$