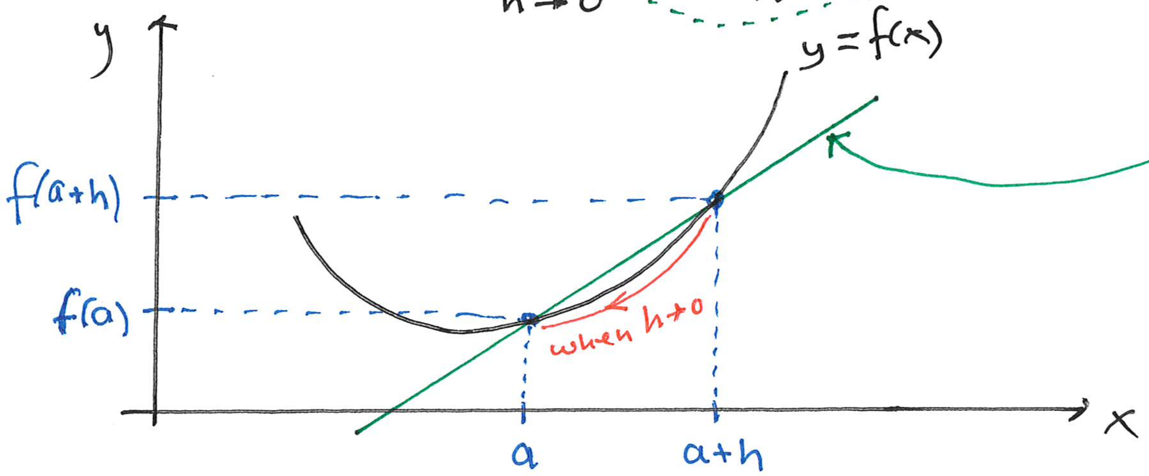
EBA 1180, lecture 17, 22 oct. 2024, Runar Ile

Plan Repetition of differentiation

1. Definition, slopes and graphs
2. The natural logarithm
3. Rules of differentiation

1. Definition, slopes and graphs

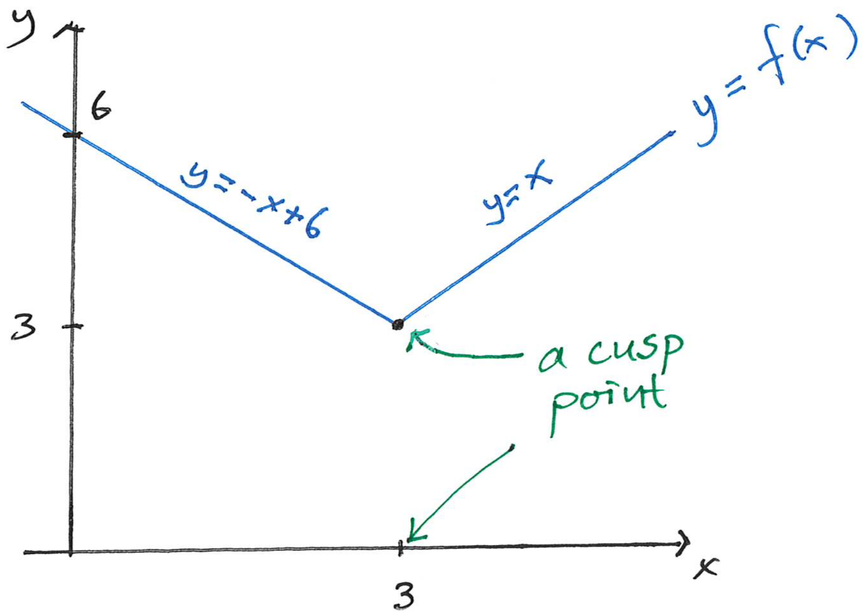
Definition: $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ — the slope of the secant



Note The derivative does not always exist!

Ex $f(x) = |x-3| + 3 = \begin{cases} -(x-3)+3 & \text{if } x < 3 \\ x-3+3 & \text{if } x \geq 3 \end{cases}$

$= \begin{cases} -x+6 & \text{if } x<3 \\ x & \text{if } x \geq 3 \end{cases}$

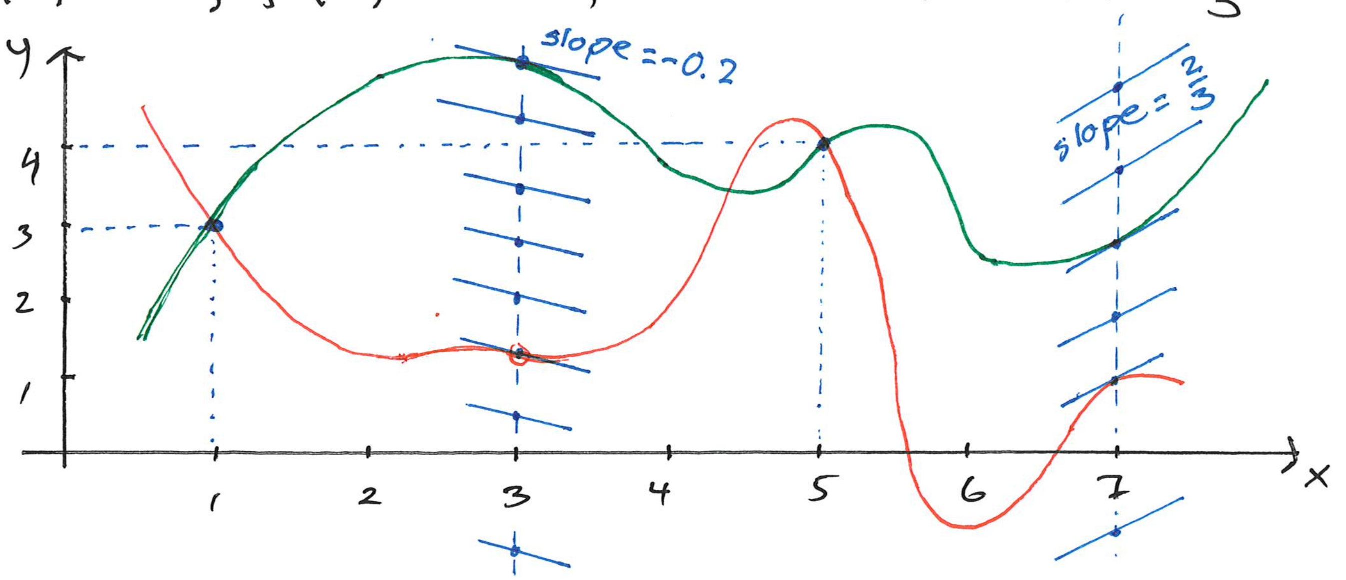


Here $f'(x) = \begin{cases} -1 & \text{if } x < 3 \\ 1 & \text{if } x > 3 \end{cases}$

But for $x = 3$ there is no tangent, hence $f'(3)$ does not exist.

Probl. 1d from last week. Sketch two graphs

$f(1) = 3$, $f'(3) = -0.2$, $f(5) = 4$, $f'(7) = \frac{2}{3}$



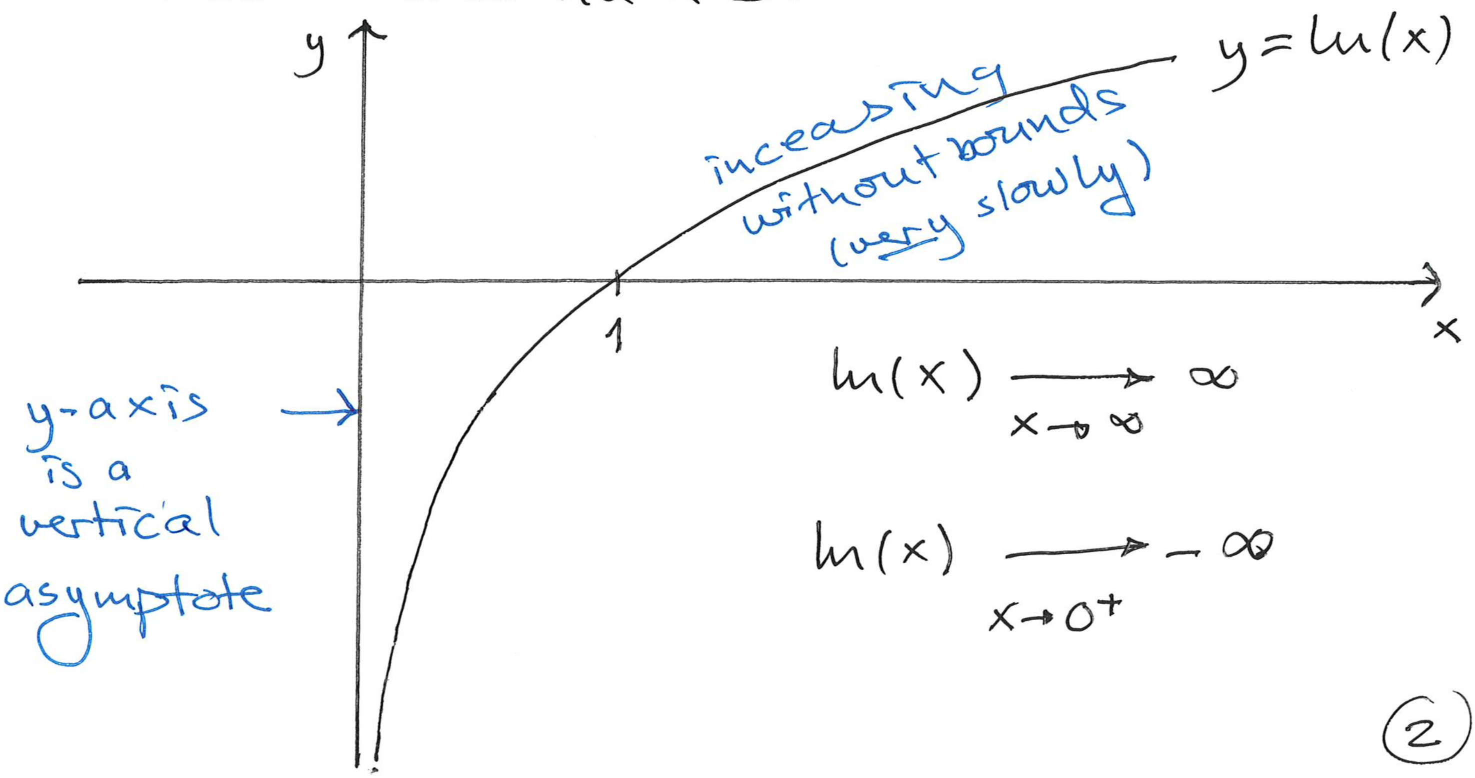
2. The natural logarithm

$\ln(x)$ is the inverse function of e^x

so $\ln(e^x) = x$ and $e^{\ln(x)} = x$.

Domain of definition of $\ln(x)$ is the range of e^x : all positive numbers

The range of $\ln(x)$ is the domain of e^x: the whole number line



$\ln(x) \xrightarrow[x \to \infty]{} \infty$

$\ln(x) \xrightarrow[x \to 0^+]{} -\infty$

Ex $\ln(\sqrt[10]{e}) = \ln(e^{\frac{1}{10}}) = \frac{1}{10}\cdot\ln(e) = \frac{1}{10}\cdot 1$

$$= \underline{\underline{\frac{1}{10}}}$$

$$\ln(3e) = \ln(3)+\ln(e) = \underline{\underline{\ln(3)+1}}$$

$$e^{2\ln(5)} = e^{\ln(5^2)} = 5^2 = \underline{\underline{25}}$$

$$\parallel$$

$$(e^{\ln(5)})^2 = (5)^2 = \underline{\underline{25}}\ .$$

$$e^{\ln(2)+\ln(3)} = e^{\ln(2)}\cdot e^{\ln(3)} = 2\cdot 3 = \underline{\underline{6}}$$

Note: $\ln(2+3) \neq \ln(2)+\ln(3)$

$$\parallel$$

$\ln(5)$ $= 0.6931 + 1.0986$

$$\parallel$$

1.6094 $= 1.7918$

different!

Ex $\ln(5x) = \ln(5)+\ln(x)$

$$\ln(x^{10}) = 10\cdot\ln(x)$$

$$\ln\left(\frac{3}{x-1}\right) = \ln(3) - \ln(x-1)$$

Start: 11.04

3. Rules of differentiation

Product rule

$$[g(x)\cdot h(x)]' = g'(x)\cdot h(x) + g(x)\cdot h'(x)$$

Ex $$[(x^2+1)e^x]' = 2x\cdot \underline{e^x} + (x^2+1)\cdot \underline{e^x}$$ common factor

$$= \underline{\underline{(x^2+2x+1)e^x}}$$ zero? $x=-1$

Ex $$[\sqrt{x}\cdot \ln(x)]' = (\underbrace{\sqrt{x}}_{=x^{\frac{1}{2}}})'\cdot \ln(x) + \underbrace{\sqrt{x}}_{=x^{\frac{1}{2}}}\cdot[\ln(x)]'$$

$$= \frac{1}{2}\cdot x^{\frac{1}{2}-1}\cdot \ln(x) + x^{\frac{1}{2}}\cdot x^{-1}$$

$$= \frac{1}{2}\underline{x^{-\frac{1}{2}}}\ln(x) + \underline{x^{-\frac{1}{2}}}$$ zero?

$$= x^{-\frac{1}{2}}\left(\frac{1}{2}\ln(x)+1\right) \qquad 1\cdot\frac{2}{2}=1$$

$$= \frac{\ln(x)+2}{2\sqrt{x}}$$

zero: $\ln(x)+2=0$

$\ln(x) = -2$

$x = e^{\ln(x)} = e^{-2}$

pos: $x > e^{-2}$

Quotient rule

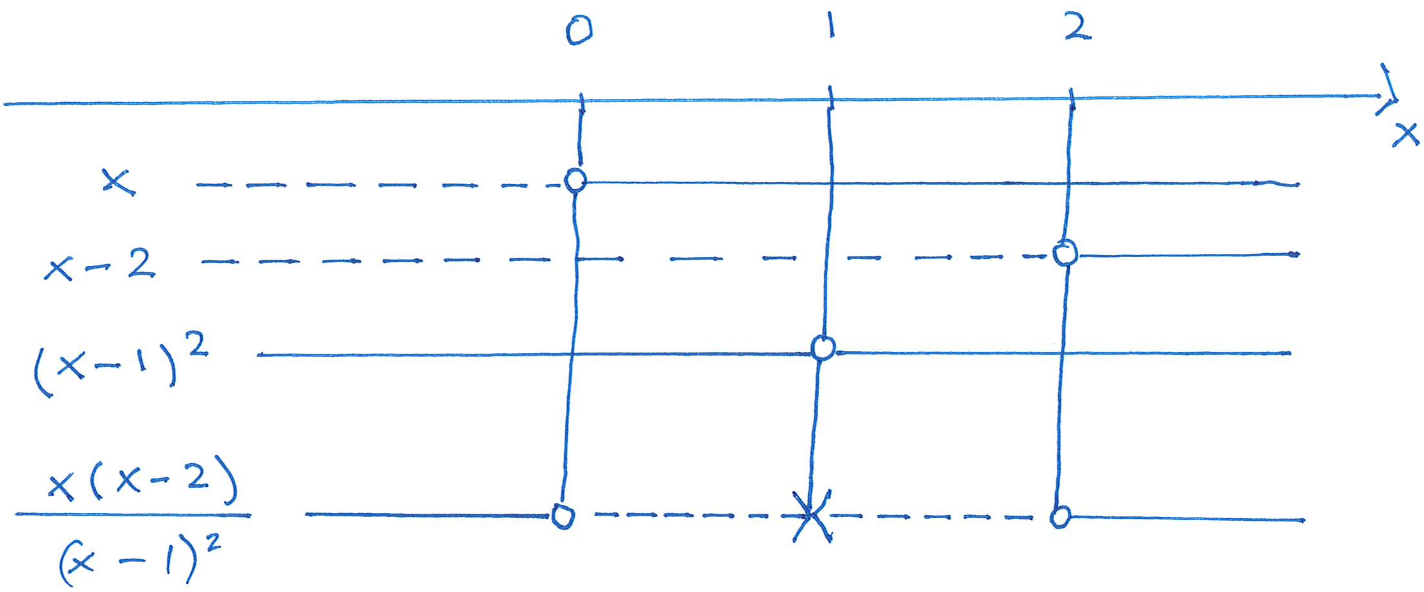
$$\left[\frac{g(x)}{h(x)}\right]' = \frac{g'(x)\cdot h(x) - g(x)\cdot h'(x)}{[h(x)]^2}$$

Ex $$\left[\frac{x^2}{x-1}\right]' = \frac{2x\cdot(x-1) - x^2\cdot 1}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$$

$$= \underline{\underline{\frac{x(x-2)}{(x-1)^2}}}$$

pos: $x>2$ or $x<0$

Sign diagram ($x = 0$, $x = 2$, $x = 1$ are zeros)



Ex $\left[\dfrac{\ln(x)}{x}\right]' = \dfrac{\frac{1}{x}\cdot x - \ln(x)\cdot 1}{x^2} = \dfrac{1 - \ln(x)}{x^2}$

zero: $x = e$

pos: $0 < x < e$

Chain rule $\left[g(u(x))\right]' = g'(u)\cdot u'(x)$

where $u = u(x)$

Ex $\left[e^{x^2+3x}\right]' = e^u \cdot (2x+3) = \underline{\underline{(2x+3)\,e^{x^2+3x}}}$

$u = u(x) = x^2+3x$ and $g(u) = e^u$

$u'(x) = 2x+3$ $\qquad g'(u) = e^u$

Ex $\left[\ln(x^2+5)\right]' = \dfrac{1}{u}\cdot 2x = \underline{\underline{\dfrac{2x}{x^2+5}}}$

$u = u(x) = x^2+5$ and $g(u) = \ln(u)$

$u'(x) = 2x$ $\qquad g'(u) = \dfrac{1}{x}$

Ex $\left[\ln\left(\frac{3x}{x-1}\right)\right]' = \left[\ln(3x) - \ln(x-1)\right]'$

$$= \left[\ln(3) + \ln(x) - \ln(x-1)\right]'$$

$$= 0 + \frac{1}{x} - \frac{1}{x-1} = \frac{x-1-x}{x(x-1)} = \frac{-1}{x(x-1)}$$

$u(x) = x-1$ and $g(u) = \ln(u)$

$u'(x) = 1$ $\qquad g'(u) = \frac{1}{u}$