EBA 1180, lecture 14, 11 Oct. 2023, Runar Ile

Plan
1. Inverse functions
2. Exponential functions
3. Logarithms

## 1. Inverse functions

What is a function?
- an expression
- a table of function values
- a graph
- a situation

Ex $f(x) = (x-3)^2$

with domain $D_f = [3, \rightarrow\rangle$

(so $x \geq 3$)

Table of function values

| $x$ | 3 | 4 | 5 | 6 | 7 | ... | $g(x)$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 1 | 4 | 9 | 16 | ... | $x$ |

← inverse function of $f(x)$.

so $g(0) = 3$, $g(1) = 4$, $g(4) = 5$ ....

$f(\overline{g(0)) = f(3) = 0}$ and $g(\overline{f(3)) = g(0) = 3}$

$f(\overline{g(1)) = f(4) = 1}$ and $g(\overline{f(4)) = g(1) = 4}$

$f(\overline{g(4)) = f(5) = 4}$ and $g(\overline{f(5)) = g(4) = 5}$

**Definition** $f(x)$ with domain $D_f$ and $g(x)$ with domain $D_g$ are inverse functions if

$f(g(x)) = x$ for all $x$ in $D_g$ and $g(f(x)) = x$ for all $x$ in $D_f$

if so, the domain of $g(x)$ is the range of $f(x)$. Short: $D_g = R_f$

Also $f(x)$ is the inverse function of $g(x)$

so $R_g = D_f$

How to find an expression for the inverse function?

(1) Solve the equation $y = f(x)$ for $x$.

(2) Switch the variables $x$ and $y$.

(3) Put $D_g = R_f$ and determine $R_f$.

Ex $f(x) = (x-3)^2$ with $D_f = [3, \rightarrow\rangle$.

We find $g(x)$ and $D_g$ by (1) - (3).

(1) Solve the equation

$$y = (x-3)^2 \text{ for all } x \text{ in } D_f$$

- take the square root on each side

$$\sqrt{y} = |x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$$

so $\sqrt{y} = x - 3$ since $x \in D_f = [3, \rightarrow\rangle$.

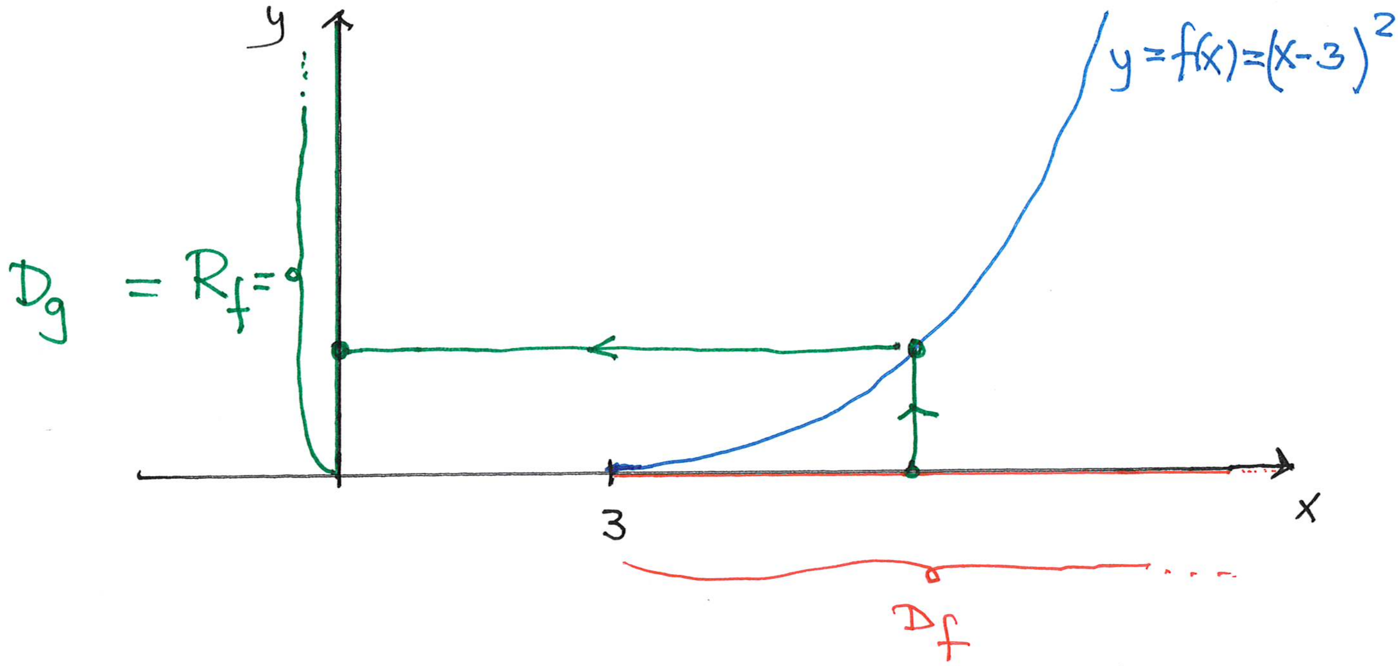
and then $x = \underline{3 + \sqrt{y}}$

(2) Switch variables : $y = g(x) = 3 + \sqrt{x}$

③ $D_g \overset{\text{always}}{=} R_f \overset{\text{claim}}{=} [0, \rightarrow\rangle$ because

$f(x) = (x-3)^2 = y$ has a solution $x$

with $x \geq 3$ for all values $y \geq 0$




Note that $f(g(x)) = f(3+\sqrt{x}) = (\underbrace{(3+\sqrt{x})-3}_{\sqrt{x}})^2 = x$

and $g(f(x)) = 3 + \sqrt{f(x)}$

$= 3 + \sqrt{(x-3)^2} \underset{\substack{\text{since}\\ x \geq 3}}{=} 3 + x - 3 = x$

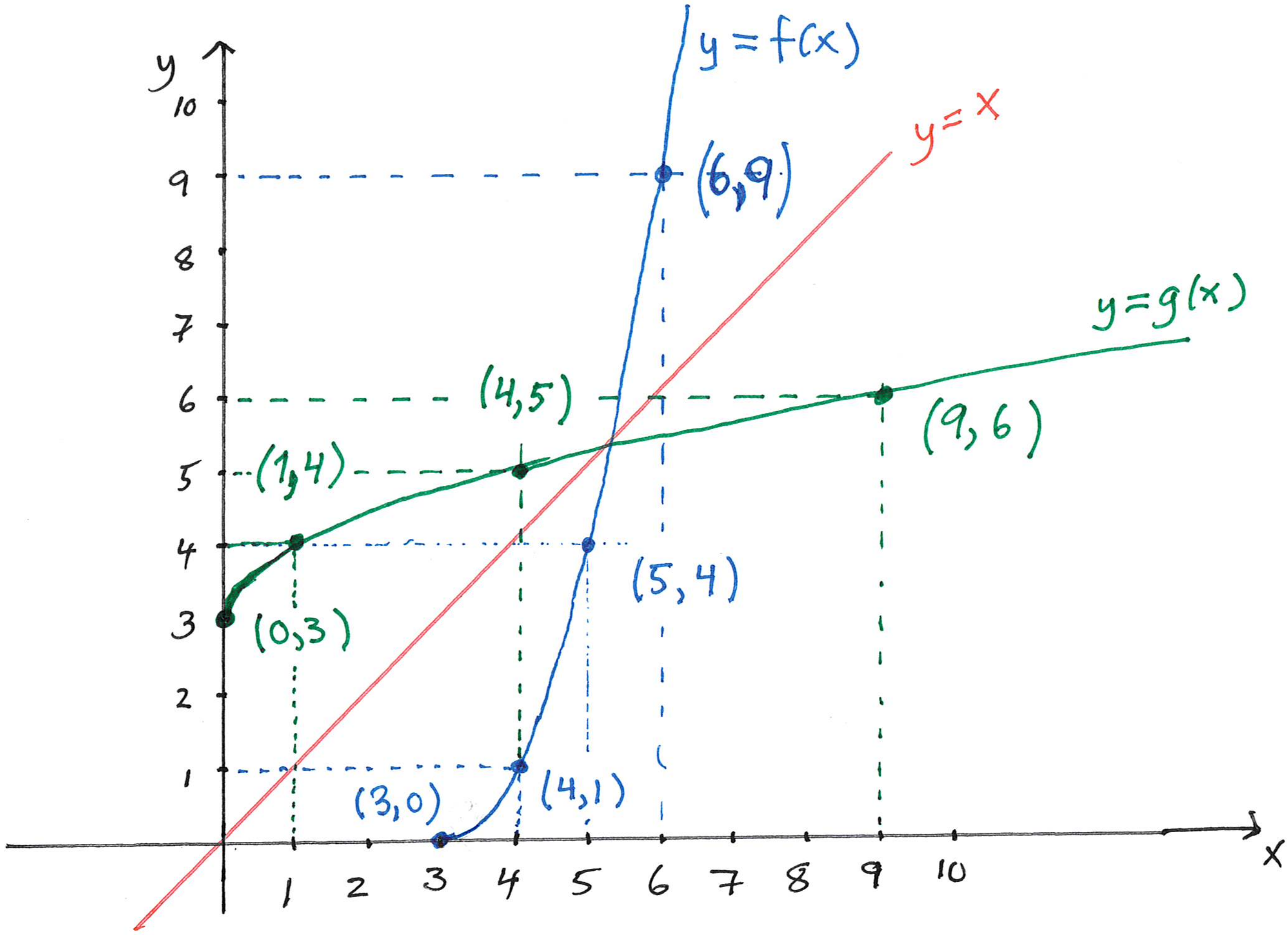
Start: 11.00

The graph of the inverse function

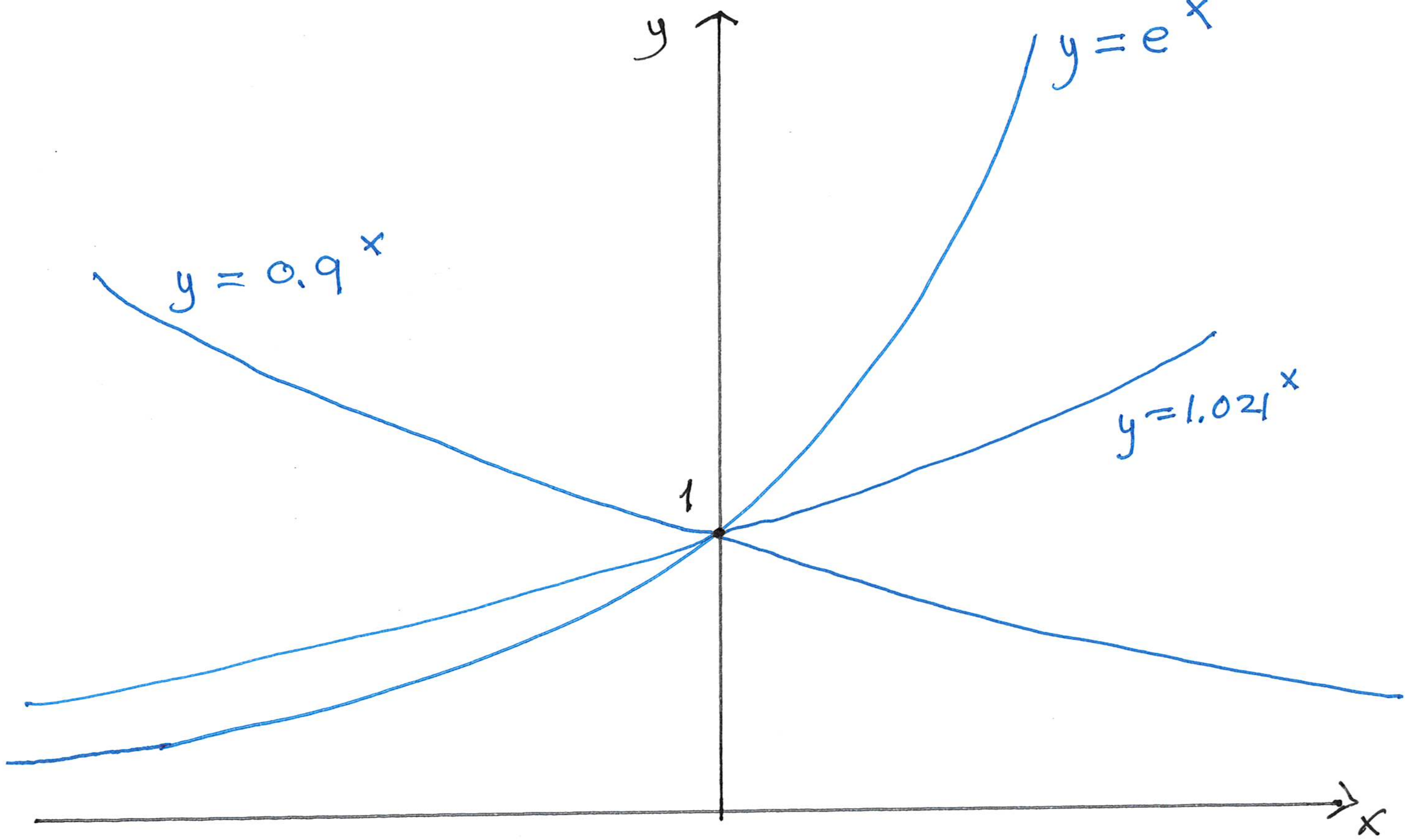
– is the mirror image of the graph of $f(x)$ with respect to the "diagonal" $y = x$

Ex $f(x) = (x-3)^2$ with $D_f = [3, \rightarrow\rangle$

| $x$ | 3 | 4 | 5 | 6 | 7 | ... | $g(x)$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 1 | 4 | 9 | 16 | ... | $x$ |




## 2. Exponential funtions



$\underline{a > 1}$ $f(x) = a^x$ is strictly increasing without upper bounds

and $a^x \xrightarrow[x \to -\infty]{} 0^+$

$\underline{0 < a < 1}$ $f(x) = a^x$ is strictly decreasing without upper bounds

<u>Both cases</u> $D_f$ = all numbers on the number line $( = \mathbb{R})$

and $R_f = \langle 0, \rightarrow \rangle$

<u>Power rules</u> If $f(x) = a^x$ then

$f(x) \cdot f(y) = a^x \cdot a^y = a^{x+y} = f(x+y)$

and $\dfrac{1}{f(x)} = \dfrac{1}{a^x} = a^{-x} = f(-x)$.

---

<u>3. Logarithms</u> Suppose $a > 0$ (and $a \neq 1$)

Then $g(x) = \log_a(x)$ is the inverse function of $f(x) = a^x$ and

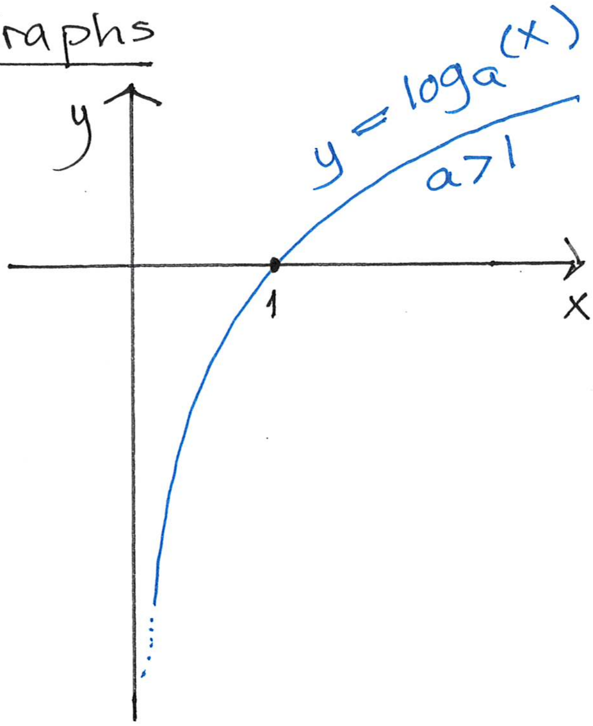
$D_g = R_f = \langle 0, \rightarrow \rangle$

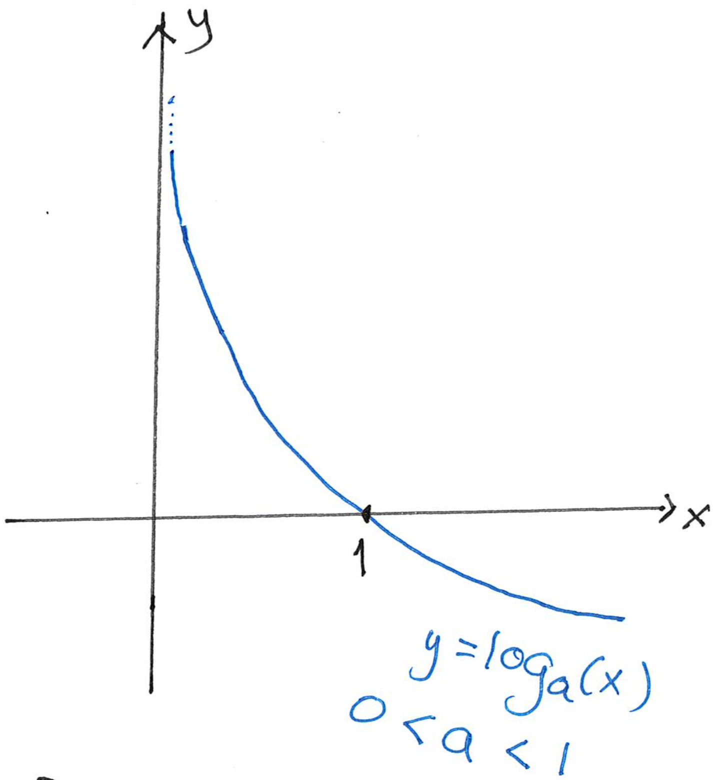
<u>EX</u> $a = 2$, $\log_2(10)$ = the number which 2 has to be raised to to give 10 (so $2^{\log_2(10)} = 10$)

We have $2^{3.322} \approx 10$

so $\log_2(10) \approx 3.322$

Graphs




the y-axis $(x=0)$ is a vertical asymptote in both cases.

Rules: $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

$$\log_a(x^r) = r \cdot \log_a(x)$$

Definition $\ln(x) = \log_e(x)$, $e$ = Euler number

- is called the natural logarithm

$\ln(x)$ is the inverse function of $e^x$

so $e^{\ln(x)} = x$ and $\ln(e^x) = x$