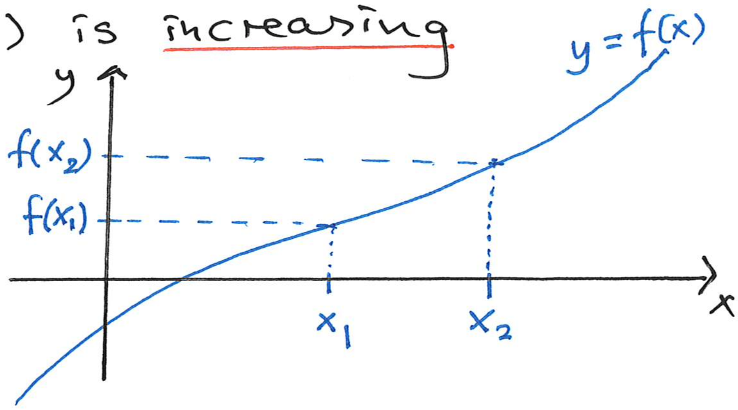
Plan
1. Increasing and decreasing functions
2. Circles and ellipses
3. Polynomial functions

1. Increasing and decreasing functions

Definition A function $f(x)$ is increasing if for all $x_1 < x_2$ one has $f(x_1) \le f(x_2)$.



Ex $f(x) = 2x + 5$ is increasing.

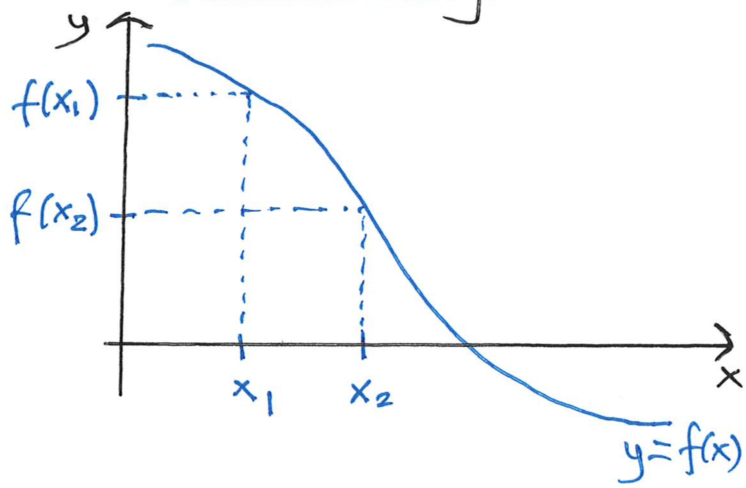
Reason: Assume $x_1 < x_2 \quad | \cdot 2$

then $2x_1 < 2x_2 \quad | + 5$

and then $2x_1 + 5 < 2x_2 + 5$

so $f(x_1) < f(x_2)$ and $f(x)$ is increasing (actually strictly increasing: $<$)

Definition A function $f(x)$ is decreasing if for all $x_1 < x_2$ one has $f(x_1) \ge f(x_2)$



Problem Show that $f(x) = -2x + 5$ is (strictly) decreasing.

Solution Suppose $x_1 < x_2 \quad | \cdot (-2)$

$$-2x_1 > -2x_2 \quad | +5$$

$$f(x_1) = -2x_1 + 5 > -2x_2 + 5 = f(x_2)$$

Conclusion : $f(x)$ is strictly decreasing.

Problem We have the constant function $f(x) = 5$. Decide whether $f(x)$ is increasing, decreasing or neither.

Solution

Increasing : If $x_1 < x_2$ then $f(x_1) = 5 \leq 5 = f(x_2)$

Decreasing : If $x_1 < x_2$ then $f(x_1) = 5 \geq 5 = f(x_2)$

- so both. But neither strictly increasing nor — " — decreasing.

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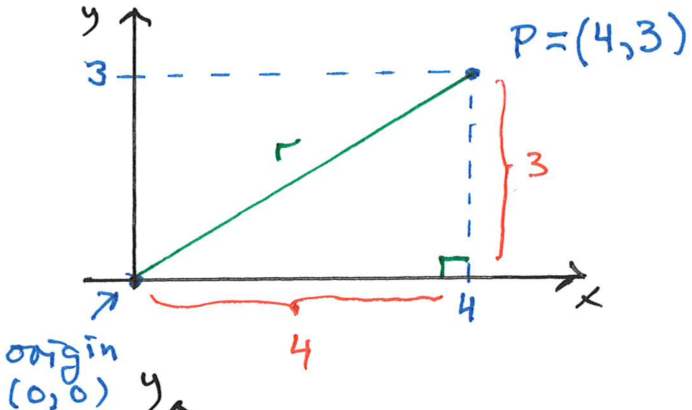
2. Circles and ellipses

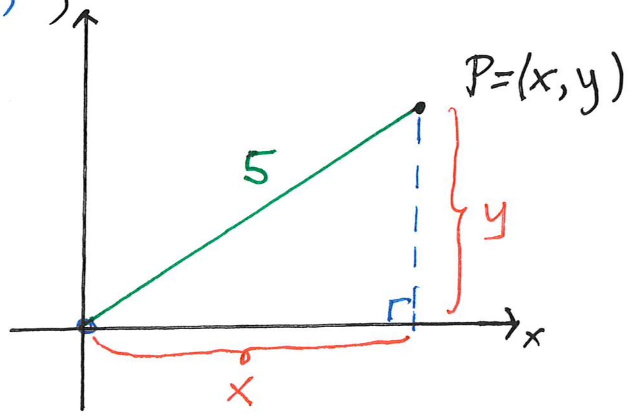


Pytagoras:

$$r^2 = 4^2 + 3^2 \quad (r \geq 0)$$

$$r^2 = 16 + 9 = 25$$

$$r = \sqrt{25} = 5$$



Pytagoras:

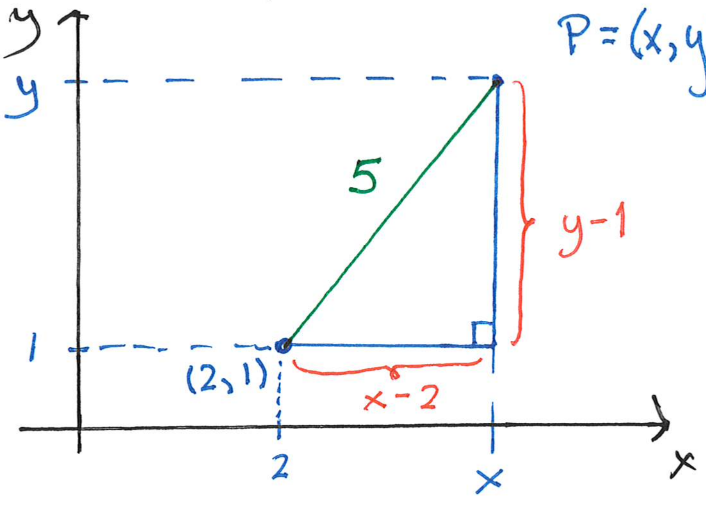
$$25 = x^2 + y^2$$

- one equation
- two unknowns
- infinitly many solutions

- The solutions are all the points on the circle with radius 5 and centre = (0,0).

Start: 11.04

Ex What is the equation for the points on a circle with radius 5 and centre (2,1)?



Pytagoras:

$5^2 = (x-2)^2 + (y-1)^2$

$25 = x^2 - 4x + 4 + y^2 - 2y + 1$

that is

$x^2 + y^2 - 4x - 2y = 20$

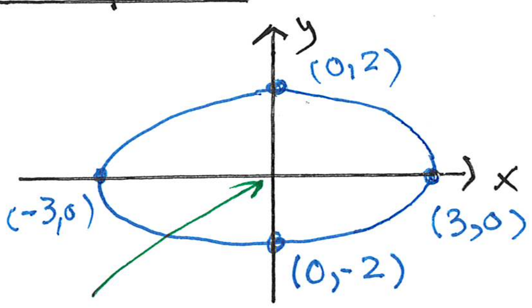
Ex Determine the radius and the centre of $x^2 + y^2 - 2x + 6y = -9$

Solution $\underbrace{(x-1)^2}_{x^2-2x+1} + \underbrace{(y+3)^2}_{y^2+6y+9} = -9+1+9 = 1$

Centre: $\underline{\underline{(1, -3)}}$ radius: $\sqrt{1} = \underline{\underline{1}}$

Ellipses $4x^2 + 9y^2 = 36$

x	3	-3	0	0	...
y	0	0	2	-2	...



the centre of the ellipse: (0, 0)

| divide each side of the eq. by 36

$\frac{4}{36} \cdot x^2 + \frac{9}{36} \cdot y^2 = 1$

$= \frac{1}{9}$ $\quad = \frac{1}{4} = \frac{1}{2^2}$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

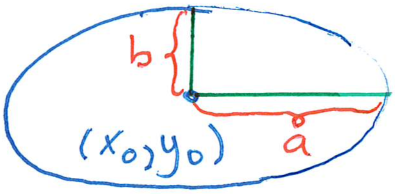
similar to a circle equation!

- have stretched the x-axis by factor 3 and y-axis by factor 2.

In general Any ellipse is the set of solutions of an equation of the form

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

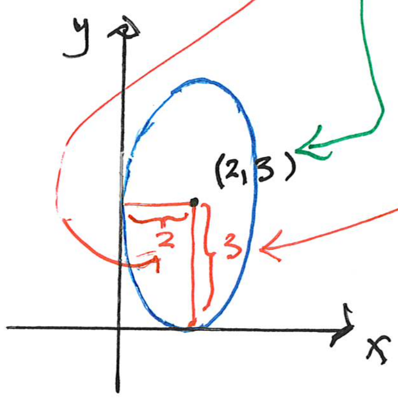
The centre is (x_0, y_0) and a and b are semi-axes.



Ex $\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$

Centre: $(2,3)$

Semi-axes: $a = \sqrt{4} = 2$, $b = \sqrt{9} = 3$



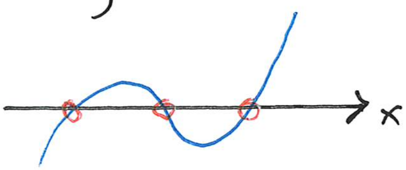
3. Polynomial functions

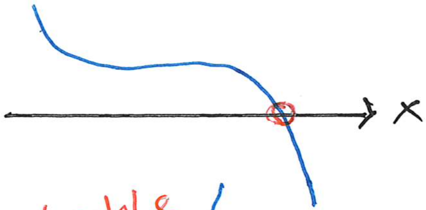
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad (a_n \neq 0)$$

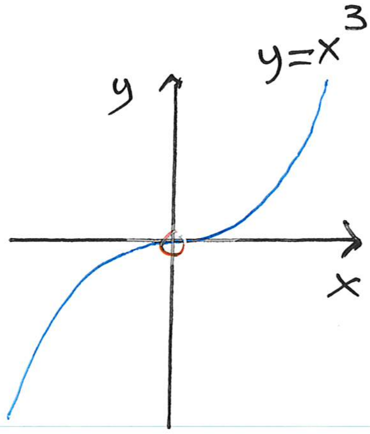
has degree n, write $\deg(f) = n$.

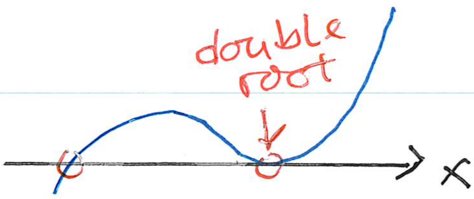
Then
- $f(x)$ has at most n roots (zeros)
- If the degree n is an odd number then $f(x)$ has at least one zero.
- If a polynomial $h(x)$ has m roots then $\deg(h) \geq m$

Ex (degree 3)

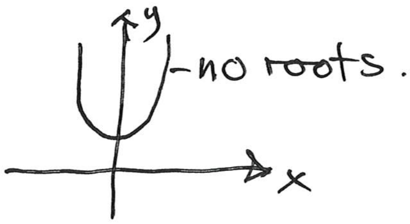


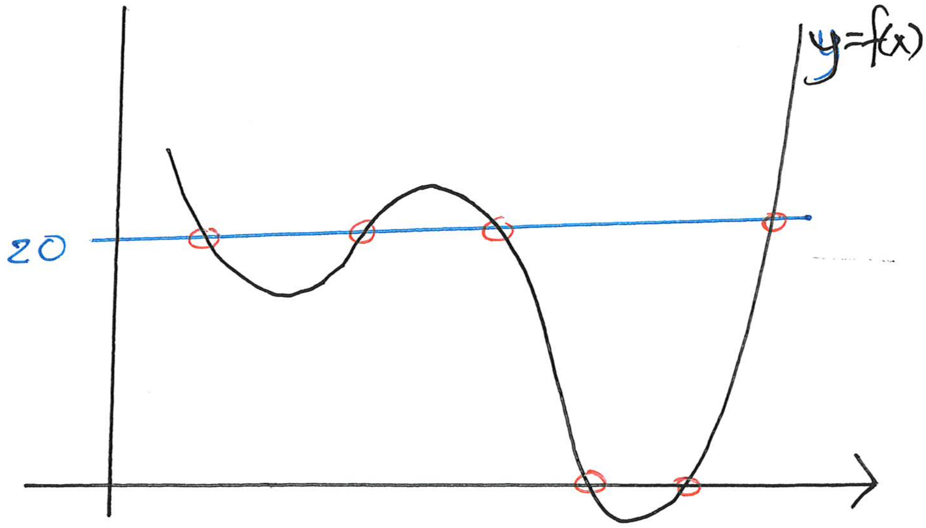






Ex $f(x) = x^4 + 1$





- only 2 roots
- but $f(x) \overset{\text{eq.}}{=} 20$ has 4 ~~root~~ solutions

that is $f(x) - 20 \overset{=0}{}$ has degree at least 4.

So $\deg(f(x)) \geq 4$.