EBA 1180, lecture 10, 25 Sept 2024, Runar Ile

Plan
1. Functions and graphs
2. Linear functions and straight lines
3. Quadratic functions and parabolas
4. Revenue and cost functions

1. Functions and graphs

Emperical functions
- the temperature as a function of time
- the price of salmon
- all kinds of 'indexes'
- fertility

Definition A function is a table of function values.

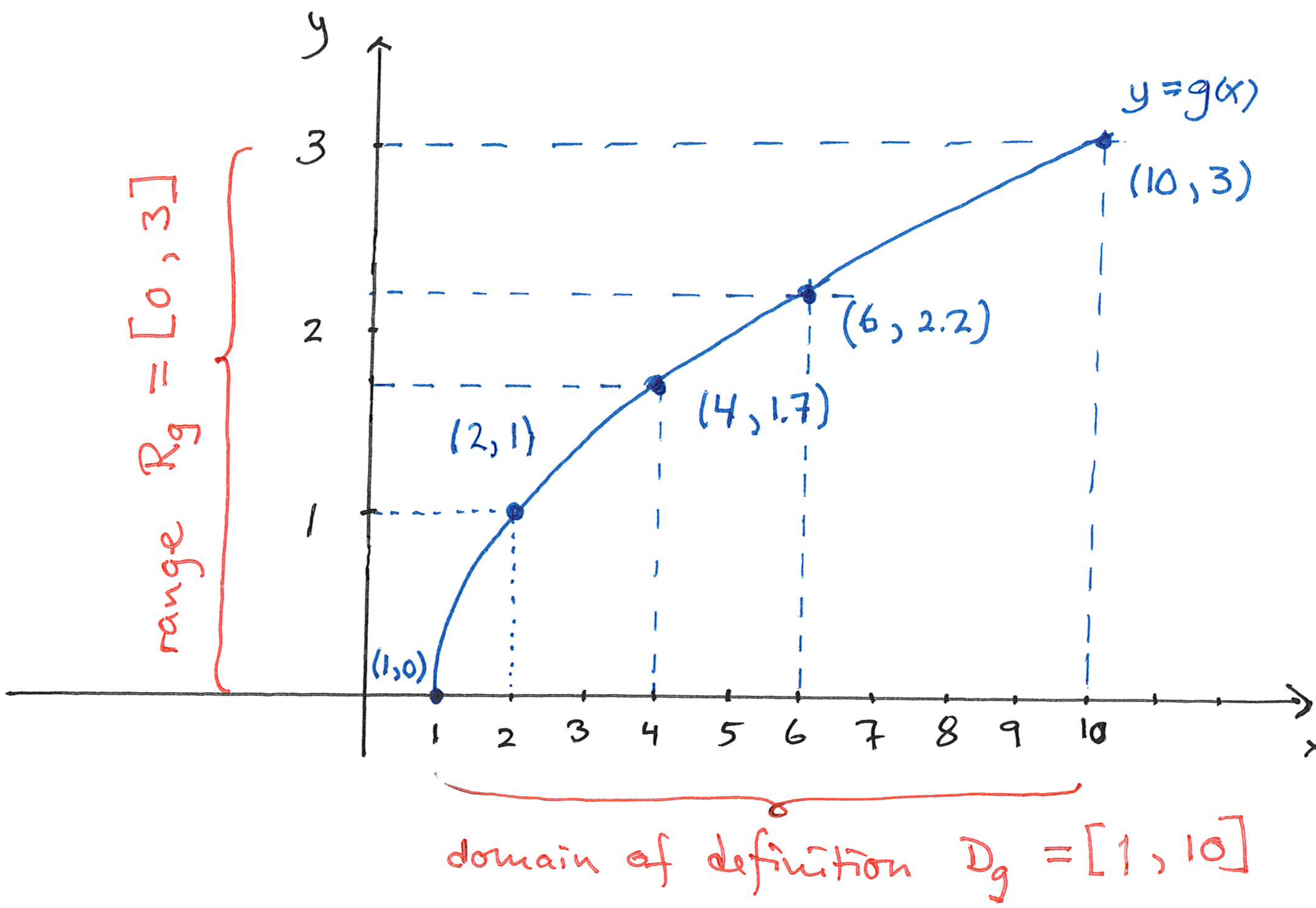
x	....
$f(x)$	....

Ex $f(x)$ = average age at the birth of the first child in year x.

Domain of definition: $x \in [1961, 2022] = D_f$

Ex $g(x) = \sqrt{x-1}$. The largest possible domain is $D_g = [1, \rightarrow\rangle$. Want to draw the graph with $D_g = [1, 10]$. I make a table of function values:

x	1	2	4	6	10
$g(x)$	0	1	1.7	2.2	3



2. <u>Linear functions</u> $\quad f(x) = ax + b$

- the graph is a line.

The point-slope formula:

If (x_0, y_0) is a point on the graph (of a line!) and a is the slope (of the line), then

$$y - y_0 = a \cdot (x - x_0)$$

dependent variable ↑ y; independent variable ↑ x

dependent variable

independent variable

Start: 11.00

Ex If $(x_0, y_0) = (9, 25)$ and $(x_1, y_1) = (11, 31)$ are two points on the line, then the slope is (the relative change)

$$a = \frac{\text{change in y-value}}{\text{change in x-value}} = \frac{31 - 25}{11 - 9}$$

$$= \frac{6}{2} = 3$$

The point-slope formula gives

$$y - 25 = 3 \cdot (x - 9)$$

so $\quad y = 3x - 27 + 25$

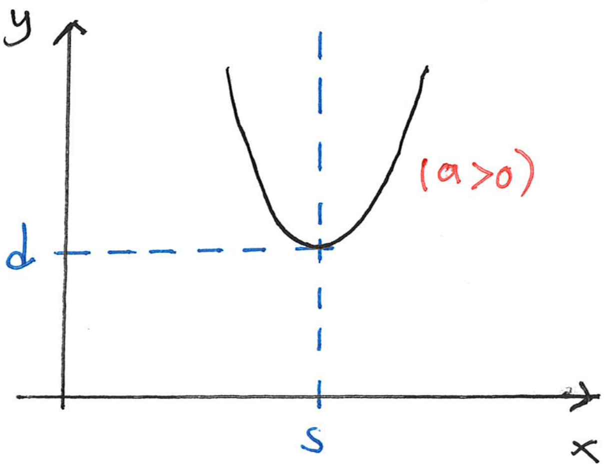
so $\quad \underline{\underline{y = 3x - 2}}$

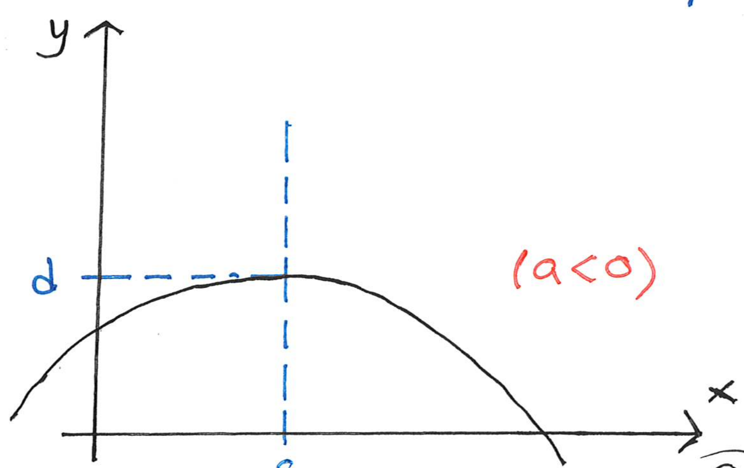
3. Quadratic functions $\quad f(x) = ax^2 + bx + c$

But if we want to draw/understand the graph, the following standard expression is better:

$$f(x) = a(x-s)^2 + d$$

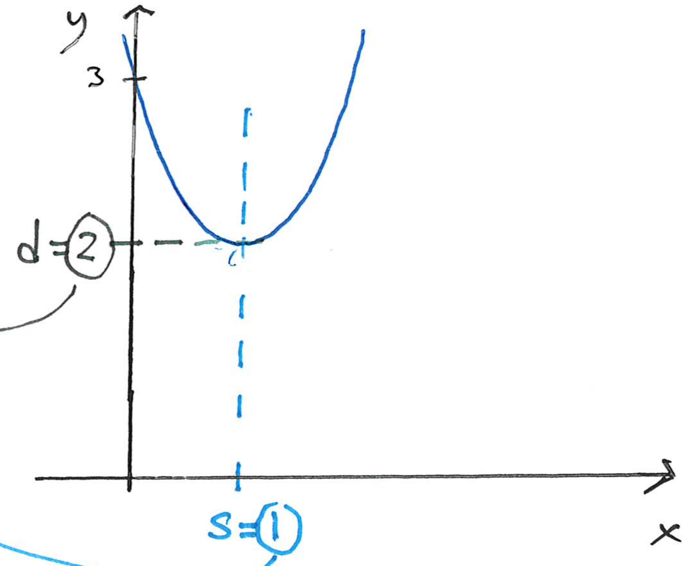
"by completing the square"





Ex $f(x) = x^2 - 2x + 3$

$= (x-1)^2 + 2$



Problem The quadratic function $f(x)$ has

- minimum value $y = -1$
- symmetry line $x = 5$ (y free)
- the graph passes through the point $(9, 3)$

a) Determine the expression $f(x) = a(x-s)^2 + d$

b) Determine where the graph crosses the x-axis and the y-axis.

Solution

a) We have been given $s = 5$ and $d = -1$

so $f(x) = a(x-5)^2 - 1$

Then $f(9) = 3$ gives the eq $a \cdot (9-5)^2 - 1 = 3$

$16a = 4$

$a = \frac{4}{16} = 0.25$

and $f(x) = \underline{\underline{0.25 \cdot (x-5)^2 - 1}}$

b) Crosses the x-axis: solve eq. $f(x) = 0$ for x.

i.e. $0.25(x-5)^2 - 1 = 0$

get $(x-5)^2 = 4$

so $x - 5 = \pm 2$ so $\underline{\underline{x = 7}}$ or $\underline{\underline{x = 3}}$

Crosse the y-axis : $y = f(0) = 0.25 \cdot (0-5)^2 - 1$

$= 0.25 \cdot 25 - 1 = 6.25 - 1 = \underline{\underline{5.25}}$

4. Revenue- and cost-functions

Profit = Revenue − cost

$P(x) = R(x) - C(x)$

x = number of units produced

$\overset{\text{assumption}}{=}$ units sold

Ex $R(x) = 15x$, $C(x) = 0.05 \cdot x^2 - 10x + 525$

Then $P(x) = 15x - (0.05x^2 - 10x + 525)$

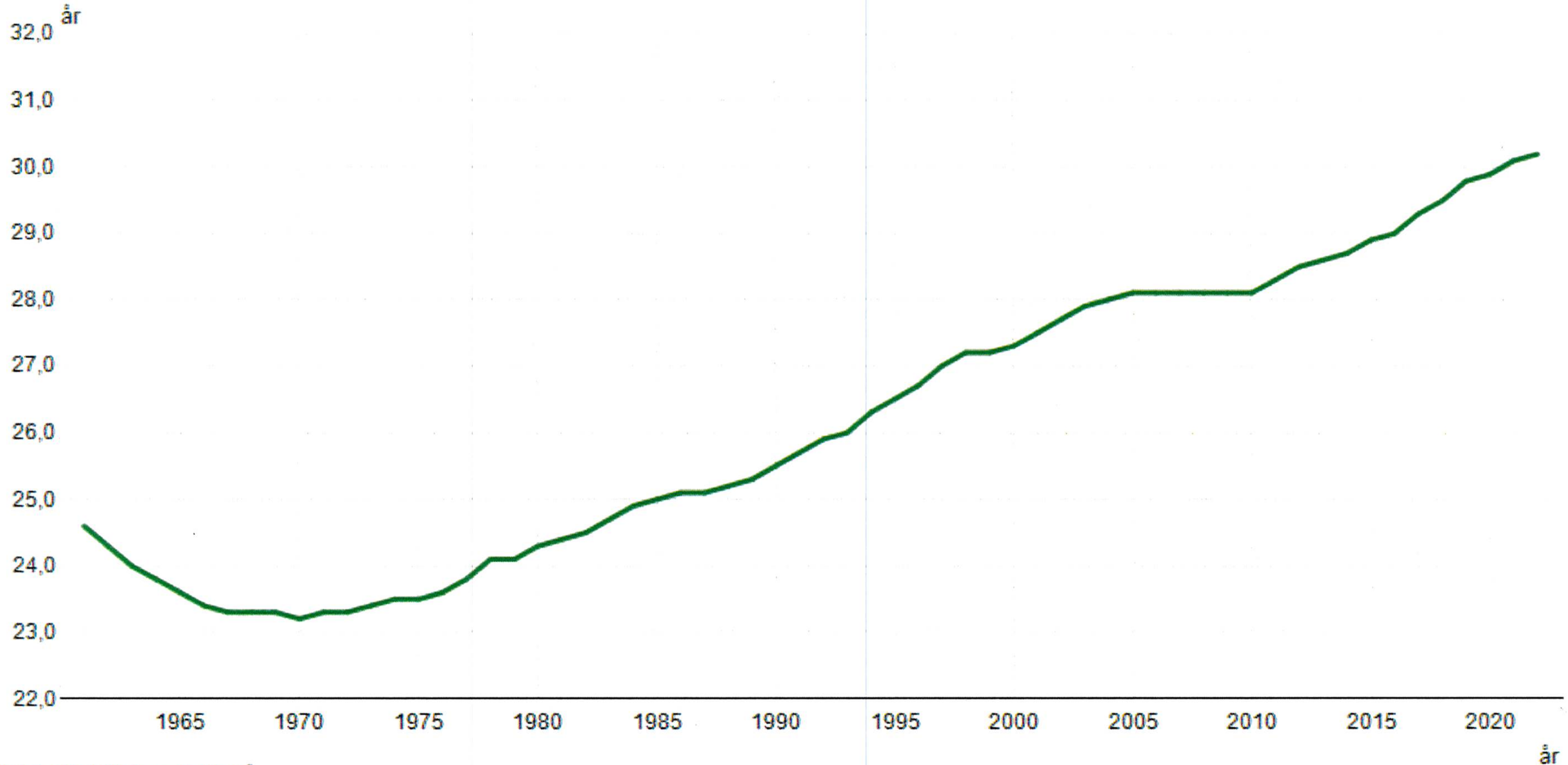
$\ldots\ldots \overset{\text{compl. sq.}}{=} -0.05\left[(x-250)^2 - 52000\right]$

So max. profit if $\underline{\underline{x = 250}}$

Max. profit $= P(250) = -0.05 \cdot (-52.000)$

$= \underline{\underline{2600}}$

07872: Foreldrenes gjennomsnittlige fødealder ved første barns fødsel, etter år. Mors fødealder første barn.

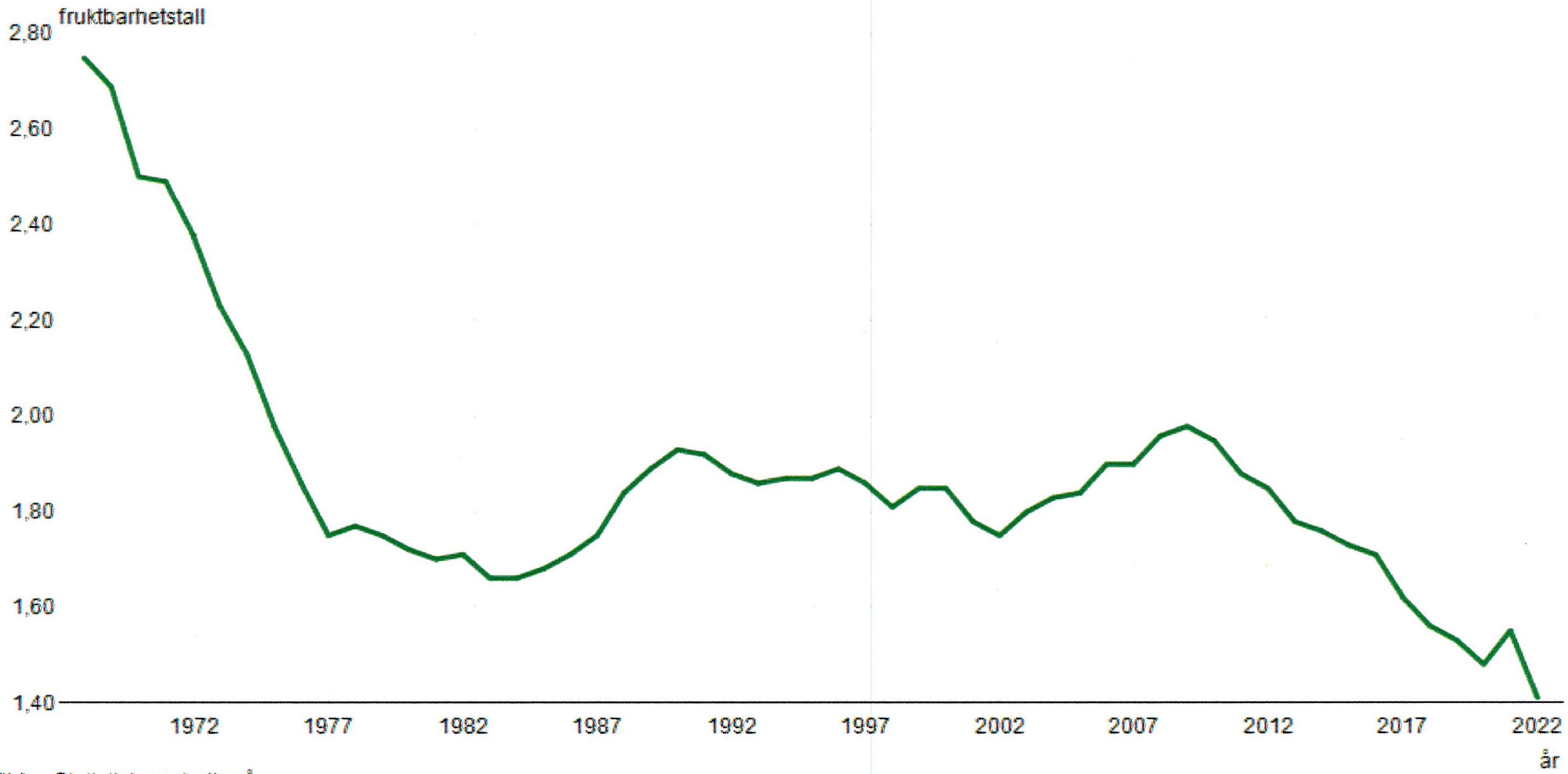


Kilde: Statistisk sentralbyrå

Fotnoter

Tall for 1961-1985 er beregnet ut fra nytt tilgjengelig datagrunnlag fra 2009. Tilsvarende datagrunnlag brukes for beregning av fars gjennomsnittsalder ved første barns fødsel.

04232: Samlet fruktbarhetstall, kvinner, etter år. Samlet fruktbarhetstall, kvinner.



Kilde: Statistisk sentralbyrå

Fotnoter

Samlet fruktbarhetstall er summen av 1-årige aldersavhengige fruktbarhetsrater 15-49 år. Antall barn hver kvinne kommer til å føde under forutsetning av at fruktbarhetsmønstret i perioden varer ved og at dødsfall ikke forekommer.

Figur 1. Barnetallfordeling 30-åringer, utvalgte kohorter. Prosent