

**EBA1180 Mathematics for Data Science**  
**autumn 2023**  
**Exercises**

*... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.*

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**Lecture 6 and 7**

**Sec. 3.1-3, 4.7:**

**Linear and quadratic equations. Equations with parametres. Polynomial equations.**

Here are recommended exercises from the textbook [SHSC].

Section 3.1 exercise 1-5

Section 3.2 exercise 2a, b, e, 4a, 5a, c

Section 3.3 exercise 1, 2, 6

**Repetition** (I have translated them, see It's Learning under «Lecture 5»)

Multiple choice exam 2017 autumn, problem 1-4

Multiple choice exam 2018 spring, problem 1, 2 and 4

**Problems for the exercise session**  
**Wednesday 13 Sept. at 12-17+ in D1-065**

**Problem 1** Solve the linear equations for  $x$ .

a)  $3x - 5 = 0$

b)  $ax + b = 0$

c)  $cx + 50 - c = 0$  ( $c \neq 0$ )

d)  $-4x + 9 = 3 - 0,25x$

e)  $ax + b = cx + d$

f)  $\frac{x}{4} + 11 - k = 3 - \frac{x}{3}$

g)  $22 - 4x + 9 - 2(x + 7) = 14 - 5(x + 2) + x - 23$

h)  $k - bx - d(3 - 5x) + 11 = c$

i)  $\frac{4}{x-2} = \frac{3}{2x+1}$

j)  $\frac{5-x}{x-a} = 7$

k)  $\frac{6}{3x+6} = \frac{14}{2x+4}$

l)  $5 - 7x = 4x + 29 - 11x$

m)  $3(x + 1) = 3x + 3$

**Problem 2** Solve the quadratic equations (without using the  $abc$ -formula).

a)  $x^2 = 0$

b)  $x^2 = 1$

c)  $x^2 = -1$

d)  $(x - 3)^2 = 0$

e)  $(x + 3)^2 = 1$

f)  $(x + 3)^2 = -1$

g)  $x^2 - 6x + 9 = 0$

h)  $x^2 - 6x + 9 = 1$

i)  $x^2 - 6x + 9 = -1$

j)  $(3x - 1)^2 = 0$

k)  $(3x - 1)^2 = 9$

l)  $(3x - 1)^2 = -9$

m)  $9x^2 - 6x + 1 = 0$

n)  $9x^2 - 6x + 1 = 9$

o)  $9x^2 - 6x + 1 = -9$

p)  $(x - 2)(x + 3) = 0$

q)  $(3x + 5)(2x - 1) = 0$

r)  $(x - 2)(x + 3) = -6$

s)  $x^2 - 5x - 2 = 0$

t)  $2x^2 + 4x = 70$

u)  $x(x - 8) = 90 - x^2$

**Problem 3** Determine the quadratic equation  $x^2 + bx + c = 0$  with the given solutions.

a)  $x = 1, x = -1$

b)  $x = 3, x = -2$

c)  $x = -3, x = -8$

d)  $x = 17, x = -3$

e)  $x = 3 \pm \sqrt{5}$

f)  $x = -11$

**Problem 4** Solve the quadratic equations by *completing the square*.

a)  $x^2 - 8x = -12$

b)  $x^2 + 10x = -9$

c)  $x^2 - 5x - 14 = 0$

d)  $x^2 - 3x = 4$

e)  $x^2 - 24x = 25$

f)  $x^2 + 5x = 6$

**Problem 5**

a) Determine  $k$  such that the equation  $x^2 - 10x = k$  has exactly one solution.

b) Determine  $k$  such that the equation  $x^2 - kx = -9$  has exactly one solution.

c) Determine  $k$  such that the equation  $\frac{1}{k}x^2 - 14x = 12$  has exactly one solution.

**Problem 6** Write the quadratic expression as a product of two linear (degree 1) polynomials.

a)  $x^2 - 8x + 12$

b)  $x^2 + 10x + 9$

c)  $x^2 - 18x + 81$

d)  $2x^2 - 4x - 70$

e)  $3x^2 + 24x - 99$

f)  $5x^2 + 60x + 180$

**Problem 7**

a) Use one parameter to write an expression for all polynomials on the form  $x^2 + bx + c$  which have two zeros of distance 1 from each other.

b) Use one parameter to write an expression for all third degree polynomials on the form  $x^3 + ax^2 + bx + c$  which have three zeros with the middle one of distance 5 to each of the other two.

c) Use two parameters to write up all quadratic polynomials with exactly one zero.

**Problem 8** Solve the equations for  $x$ .

a)  $(2x - \sqrt{3})(x^2 - 20x + 99) = 0$

b)  $(x^2 - 5)(x^2 - 6x + 4) = 0$

c)  $(2x^2 - 28x + 98)(x^2 - 6x + 10)(x^2 - 2x - 6) = 0$

d)  $(x + 1)(x + 2)(x + 3)(x + a)(x^2 + b) = 0$

**Problem 9** Solve the equations.

a)  $x^4 - 11x^2 + 18 = 0$

b)  $x^6 + 13x^3 + 40 = 0$

c)  $x^{100} - x^{50} - 12 = 0$

**Problem 10** Solve the equations.

a)  $1 + x + x^2 + x^3 + \dots + x^{98} + x^{99} = 0$

b)  $1 + x + x^2 + x^3 + \dots + x^{49} + x^{50} = 0$

c)  $\frac{x}{1.02} + \frac{x^2}{1.02^2} + \frac{x^3}{1.02^3} + \frac{x^{29}}{1.02^{29}} + \frac{x^{30}}{1.02^{30}} = 0$

## Answers

### Problem 1

- a)  $x = \frac{5}{3}$                       b)  $x = -\frac{b}{a}$                       c)  $x = \frac{c-50}{c} = 1 - \frac{50}{c}$                       d)  $x = \frac{8}{5}$
- e)  $x = \frac{d-b}{a-c}$  if  $a \neq c$ . For  $a = c$  and  $b \neq d$  there is no solution,  
for  $a = c$  and  $b = d$  all numbers are solutions.
- f)  $x = \frac{12k-96}{7}$                       g)  $x = 18$
- h)  $x = \frac{c+3d-k-11}{5d-b}$  if  $b \neq 5d$ . If  $b = 5d$  and  $c + 3d \neq k + 11$  there are no solutions,  
if  $b = 5d$  and  $c + 3d = k + 11$  all numbers are solutions.
- i)  $x = -2$                       j)  $x = \frac{7a+5}{8}$                       k) no solutions                      l) no solutions
- m) all numbers are solutions

### Problem 2

- a)  $x = 0$                       b)  $x = 1, x = -1$                       c) no solutions                      d)  $x = 3$
- e) either  $x + 3 = 1$ , i.e.  $x = -2$ ,  
or  $x + 3 = -1$ , i.e.  $x = -4$                       f) no solutions                      g)  $x = 3$
- h)  $x = 2, x = 4$                       i) no solutions                      j)  $x = \frac{1}{3}$                       k)  $x = \frac{1}{3} \pm 1$ , i.e.  
 $x = \frac{4}{3}, x = -\frac{2}{3}$
- l) no solutions                      m)  $x = \frac{1}{3}$                       n)  $x = \frac{4}{3}, x = -\frac{2}{3}$                       o) no solutions
- p)  $x = 2, x = -3$                       q)  $x = -\frac{5}{3}, x = \frac{1}{2}$                       r)  $x = 0, x = -1$                       s)  $x = \frac{5}{2} \pm \frac{\sqrt{33}}{2}$
- t)  $x = -7, x = 5$                       u)  $x = -5, x = 9$

### Problem 3

- a)  $(x-1)(x-(-1)) = x^2 - 1$                       b)  $x^2 - x - 6$                       c)  $x^2 + 11x + 24$
- d)  $x^2 - 14x - 51$                       e)  $x^2 - 6x + 4$                       f)  $x^2 + 22x + 121$

### Problem 4

- a)  $(x-4)^2 = -12 + 16$  gives  
 $x = 2, x = 6$                       b)  $(x+5)^2 = -9 + 25$  gives  
 $x = -9, x = -1$                       c)  $(x-\frac{5}{2})^2 = 14 + \frac{25}{4}$  gives  
 $x = -2, x = 7$
- d)  $(x-\frac{3}{2})^2 = 4 + \frac{9}{4}$  gives  
 $x = -1, x = 4$                       e)  $(x-12)^2 = 25 + 12^2$  gives  
 $x = -1, x = 25$                       f)  $(x+\frac{5}{2})^2 = 6 + \frac{25}{4}$  gives  
 $x = -6, x = 1$

### Problem 5

- a)  $k = -25$                       b)  $k = \pm 6$                       c)  $k = -\frac{12}{49}$

### Problem 6

- a)  $(x-2)(x-6)$                       b)  $(x+9)(x+1)$                       c)  $(x-9)^2$
- d)  $2(x+5)(x-7)$                       e)  $3(x+11)(x-3)$                       f)  $5(x+6)^2$

**Problem 7**

- a) If  $r$  is the smallest zero, the polynomial is  $x^2 - (2r + 1)x + r(r + 1)$ .  
b) If  $r$  is the second smallest zero, the polynomial is  
 $(x - r)^3 - 25(x - r) = x^3 - 3rx^2 + (3r^2 - 25)x - (r - 5)r(r + 5)$ .  
c)  $a(x - r)^2 = ax^2 - 2arx + ar^2$  ( $a \neq 0$ ).

**Problem 8**

- a)  $x = \frac{\sqrt{3}}{2}$ ,  $x = 9$ ,  $x = 11$   
b)  $x = \pm\sqrt{5}$ ,  $x = 3 \pm \sqrt{5}$   
c)  $x = 7$ ,  $x = 1 \pm \sqrt{7}$   
d)  $x = -1$ ,  $x = -2$ ,  $x = -3$ ,  $x = -a$ ,  $x = \pm\sqrt{-b}$  for  $b \leq 0$

**Problem 9**

- a)  $x = \pm\sqrt{2}$ ,  $x = \pm 3$  (Hint: Substitute  $u = x^2$ )  
b)  $x = -2$ ,  $x = -\sqrt[3]{5}$   
c)  $x = \pm\sqrt[25]{2}$

**Problem 10**

- a)  $x = -1$   
b) no solutions  
c)  $x = 0$ ,  $x = -1.02$